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Functions in Mathematica (a selection) (MathFunctionsinMMA.pdf)

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Important Note

The author reserves all rights on this manuscript.
The manuscript was prepared by use of Mathematica,
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This manuscript contains the sheets or notebooks given to the participants attending the course on Symbolic Computation. It contains almost all the commands, examples and results. But, for lack of time and space, these sheets do not contain all the explanations and comments given orally. So for an uninitiated reader it may be difficult to grasp the meaning of some commands and the reasons why particular examples are presented. In many cases the reader may find such comments by looking up the commands in the Table at the end of the Wolfram's Mathematica book and following up the references listed therein. I intend to include more comments later if my duties will permit me to do so.

The idea of the lecture and of the manuscript is to show how a given mathematical problem or task encountered in Theoretical Physics or Applied Mathematics may be solved within the framework of Mathematica. The user may look up a particular subject in the Table of Contents. It was my intention to render the corresponding descriptions as self-contained as possible. So a beginner, who is familiar with starting a Mathematica session on his computer, may begin with Chapters 2, 3 and 4, grasp a cursory knowledge of Chapter 5 and then jump right into the Chapter containing the subject he is interested in. Chapters 20 to 23 contain material going into some depths and require some knowledge from earlier chapters.

I should be very grateful to any reader or user, who is willing to point out missprints, errors, omissions, inconsistencies and unclear presentations. I suggest to write the corrections or remarks on the sheet containing the text in question and to send me a copy. or to communicate by any other means.

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