# **Functions in Mathematica (A Selection)**

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**Numerical Functions** (§ 13.2)

Floor[x] Ceiling [x] Round[x] Chop[x] Max[] Min[] N[]

Complex Numbers (§ 13.3)

Re[] Im[] Abs[] Arg[] Conjugate[]

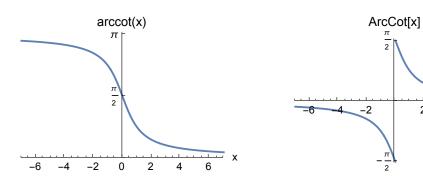
Random Numbers (§ 13.4)

Random[]

### functionTrigonometric Functions and Inverses (§ 13.9)

Sin[z]	ArcSin[z]	Cot[z] = 1/Tan[z]	ArcCot[z]
Cos[z]	ArcCos[z]	Csc[z] = 1/Sin[z]	ArcCsc[z]
Tan[z]	ArcTan[z]	Sec[z] = 1/Cos[z]	ArcSec[z]

The range of values of the function program ArcCot[z] disagreßes with the ususal definition of the principal branch of the multi-valued function arccot[x], s. Fig.



Exponentials, Logarithms (§ 13.8)

Exp[z] Log[z] (Basis e) Log[b,z] (Basis b), e.g. Log[10,z]

## **Hyperbolic Functions and Inverses** (§ 13.10)

Cosh[z]	$ArcCosh[z] = Log[z + \sqrt{(z^2 - 1)}]$
Sinh[z]	$ArcSinh[z] = Log[z + \sqrt{(z^2 + 1)}]$
Tanh[z]	ArcTanh[z] = Log[(1+z)(1-z)]/2
Coth[z] = 1/Tanh[z]	ArcCoth[z] = Log[(z+1)(z-1)]/2
Csch[z] = 1/Sinh[z]	$ArcCsch[z] = Log[1/z + \sqrt{(1/z^2 + 1)}]$
Sech[z] = 1/Cosh[z]	$ArcSech[z] = Log[1/z + \sqrt{(1/z^2 - 1)}]$

### Factorials, Binomial Coefficients and Vector Coupling Coefficients (§ 13.5)

$$\begin{split} z! &= Gamma[z+1] & ClebschGordan[\{j1,m1\},\{j2,m2\},\{j3,m3\}] \\ n!! &= n \ (n-2)() \ ... \ 2 \ or \ 1 & ThreeJSymbol[\{j1,m1\},\{j2,m2\},\{j3,m3\}] \\ Binomial[z,k] &= z!/[k! \ (n-k)!] & SixJSymbol[\{j1,j2,j3\},\{j4,j5,j6\}] \end{split}$$

#### **Error Functions**

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$
,  $\operatorname{Erfc}[z] = \operatorname{erfc}(z) = 1 - \operatorname{erf}(z)$ .

### **Spherical Harmonics and Legendre Polynomials**

$$\begin{aligned} \text{LegendreP}[n,z] &= P_n(z) & \text{LegendreP}[n,m,z] &= P_n \stackrel{m}{(z)} \\ \text{LegendreQ}[n,z] &= Q_n(z) & \text{LegendreQ}[n,m,z] &= P_n \stackrel{m}{(z)} \end{aligned}$$

SphericalHarmonicY[m,n,theta,phi] =  $Ynm(\vartheta,\phi)$ 

# **Legendre Functions**

Solutions of Legendres differential equation for values oft he parameters leading to transcendental functions. Special cases are Toroidal functions and Conical functions.

# Hermite and Laguerre Polynomials

$$\begin{aligned} & \text{HermiteH}[n,z] = H_n(z) & \text{LaguerreL}[n,z] = L_n[z] \\ & \text{LaguerreL}[n,\alpha,z] = L_n^{\alpha}[z] \end{aligned}$$

### **Cylindrical functions = Besselfunctions**

BesselJ[
$$v,z$$
] = J <sub>$v$</sub> ( $z$ )

BesselY[ $v,z$ ] = Y <sub>$v$</sub> ( $z$ )

BesselK[ $v,z$ ] = K <sub>$v$</sub> ( $z$ )

#### **Mathieu functions**

Elliptic Cylinder functions. Some of the functions programmes do not give correct values for some values of the parameters.

#### **Spheroidal functions**

Solutions for the Helmholtz equation in rotational elliptic coordinates

#### **Hypergeometric functions**

(§ 7.2.2)

Almost all functions are defined as given in the books listed below. Note that Log, inverse trigonometric and inverse hyperbolic functions are multiple-valued. The programs give the principal value. This requires special care, in particular, in using results of analytical integrations.

There is one important **exception**: *Mathematica* defines **the principal value** of **arccot**[x] in a way differnt from the usual one.

In most cases, *Mathematica* uses the functions as defined in

#### **Books on Functions:**

M. Abramowitz and I. Stegun: Handbook of Mathematical Functions, Verlag Dover, 1974 Poceketbook of Mathematical Functions, Verlag Deutsch, 1984

F.W.J. Olver, D.W. Lozier, R.F. Boisvert, Ch.W. Clark: NIST Handbook of Mathematical Functions Cambridge University Press, 2010

#### List of functions and their properties:

An extensive list of properties of many functions is at: http://functions.wolfram.com