| c) Elektrostatik: Endliche Ladungsverteilung im homogenen Raum. | | d) An den Enden eingespannte Saite der Länge $\ell$. |
|---|---|---|
| $\Delta\Phi = -\rho(\vec{r})/\varepsilon_o$ | (1a) | $d^2\phi/dx^2 + k^2\,\phi(x) = -g_0(x)$ |
| $\lim_{\lvert\vec{r}\rvert\to\infty} \Phi(\vec{r}) = 0$ | (1b) | $\phi(x=0) = \phi(x=\ell) = 0.$ |
| Gegeben: $\Delta$, $\rho(\vec{r})$, $\varepsilon$. | | Gegeben: $d^2/dx^2$, $g_0(x)$, $k^2$, $\ell$. |
| Es gibt keinen Eigenwert. | (2) | $k \neq k_n = n\pi/\ell, \quad n \in \mathbb{N}.$ |
| $\Delta G(\vec{r},\vec{r}\,') = -\delta(\vec{r}-\vec{r}\,')$ | (3a) | $d^2G/dx^2 + k^2\,G(x,x') = -\delta(x-x')$ |
| $\lim_{\lvert\vec{r}\rvert\to\infty} G(\vec{r},\vec{r}\,') = 0$ | (3b) | $G(x=0,x') = G(x=\ell,x') = 0.$ |
| $\Phi(\vec{r}) = \iiint d\vec{r}\,' \frac{1}{4\pi\lvert\vec{r}-\vec{r}\,'\rvert}\frac{\rho(\vec{r}\,')}{\varepsilon}$ $= \iiint d\vec{r}\,'\,G(\vec{r},\vec{r}\,')\,\rho(\vec{r}\,')/\varepsilon$ | (4) | $\phi(x) = \int_0^\ell dx'\,G(x,x')\,g_0(x')$ |
| $G(\vec{r},\vec{r}\,') = \frac{1}{4\pi}\frac{1}{\lvert\vec{r}-\vec{r}\,'\rvert}$ | (5) | $G(x,x') = \frac{2}{\ell}\sum_{n=1}^{\infty}\frac{\sin(k_n x)\,\sin(k_n x')}{k^2-k_n^2}$ |
| $\rho(\vec{r}) = 0 \Rightarrow \Phi(\vec{r}) = 0.$ | (6) | $g_0(x) = 0 \Longrightarrow \phi(x) = 0.$ |
| | (7) | $k = k_s = s\pi/\ell\,, \quad s \in \mathbb{N}.$ |
| | (8) | $d^2\phi_s/dx^2 + k_s^2\phi_s(x) = 0.$ $\phi_s(0) = \phi_s(\ell) = 0$ |
| | (9) | $(\phi_s, g_0) = \int_0^\ell \phi_s(x)\,g_0(x)\,dx = 0$ |
| | | Greensche Funktion in verallg.Sinn: |
| | (10) | $G(x,x') = \frac{2}{\ell}\sum_{\substack{n=1\\ n\neq s}}^{\infty}\frac{\sin(k_n x)\,\sin(k_n x')}{k_s^2-k_n^2}$ |
| 2. Greenscher Satz: | | |
| $u\Delta v - v\Delta u = \mathrm{div}\,\vec{j}$ $\vec{j} = u\nabla v - v\nabla u$ | (11) | $u\,d^2v/dx^2 - v\,d^2u/dx^2 =$ $d/dx(u\,dv/dx - v\,du/dx)$ |
| $\iiint_V (u\Delta v - v\Delta u)\,dr =$ $= \iint_F dF\,(u\,\partial v/\partial n - v\,\partial u/\partial n)$ | (12) | $\int_0^\ell (u\,d^2v/dx^2 - v\,d^2u/dx^2)\,dx =$ $= (u\,dv/dx - v\,du/dx)\big\rvert_{x=o}^{x=\ell}$ |