$$\Delta G + k^2 G = -\delta(\vec{r}-\vec{r}\,') \qquad e^{-i\omega t}$$ ②

$$k = k_1 + i k_2, \qquad k_1 \geq k_2 \geq 0.$$

RB: Ausstrahlungsbedingung im Unendlichen.

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2} + k^2\right] G(r,\varphi,z;r',\varphi',z') = -\frac{\delta(r-r')}{r}\,\delta(\varphi-\varphi')\,\delta(z-z')$$

$$\delta(z-z') = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\zeta\, e^{i\zeta(z-z')}$$

$$\delta(\varphi-\varphi') = \frac{1}{2\pi}\sum_{m=-\infty}^{\infty} e^{im(\varphi-\varphi')}$$

$$G(r,\varphi,z;r',\varphi',z') = \frac{1}{4\pi^2}\sum_{m=-\infty}^{\infty} e^{im(\varphi-\varphi')}\int_{-\infty}^{\infty} d\zeta\, e^{i\zeta(z-z')}\, g_m(r,r';\zeta)$$

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{m^2}{r^2} \underbrace{-\zeta^2 + k^2}_{\lambda^2}\right] g_m(r,r;\zeta) = -\frac{\delta(r-r')}{r}$$

$$\lambda^2 = k^2 - \zeta^2; \quad \lambda = \lambda_1 + i\lambda_2 = \sqrt{k^2-\zeta^2}, \quad \mathrm{Re}(\lambda) \geq 0,\ \mathrm{Im}(\lambda) \geq 0$$

längs Integrationsweg

part. Lösungen: $J_m(\lambda r),\ H_m^{(1)}(\lambda r) \sim e^{i\lambda r} = e^{i\lambda_1 r}\, e^{-\lambda_2 r}$

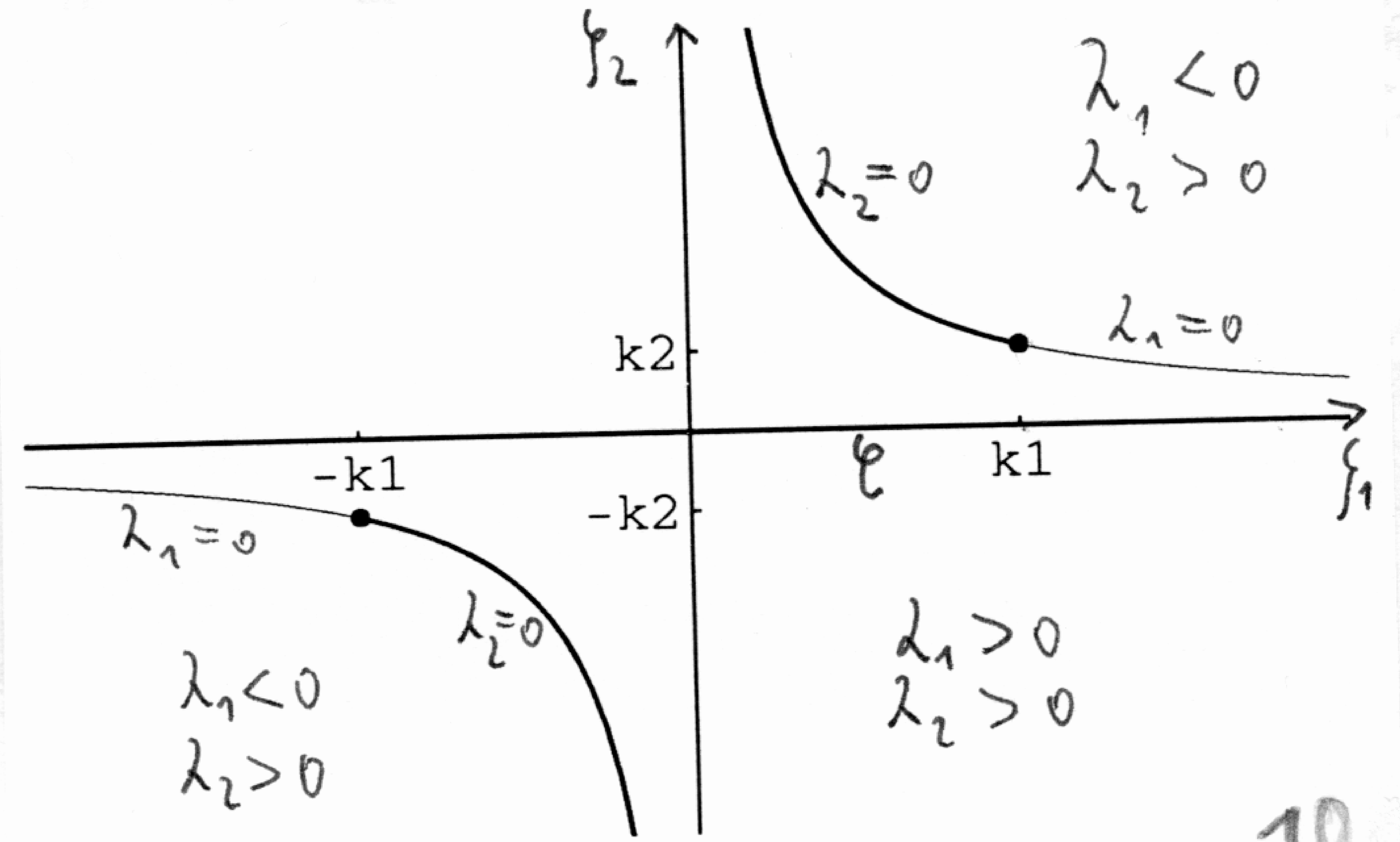
$$\frac{d}{dr}\left(r\frac{d}{dr}\right) g_m + \left(-\frac{m^2}{r} + \lambda^2\right) g_m = -\delta(r-r')$$

$$g_m = A_m J_m(\lambda r_<)\, H_m^{(1)}(\lambda r_>)$$

$$p(r) = r, \quad W = \lambda \begin{vmatrix} J_m(\lambda r') & H_m^{(1)}(\lambda r') \\ J_m'(\lambda r') & H_m^{(1)\prime}(\lambda r') \end{vmatrix} = \lambda \frac{-2}{\pi i \lambda r'} = \frac{2i}{\pi r'}, \qquad A_m = -\frac{1}{pW} = -\frac{\pi}{2i} = \frac{i\pi}{2}$$

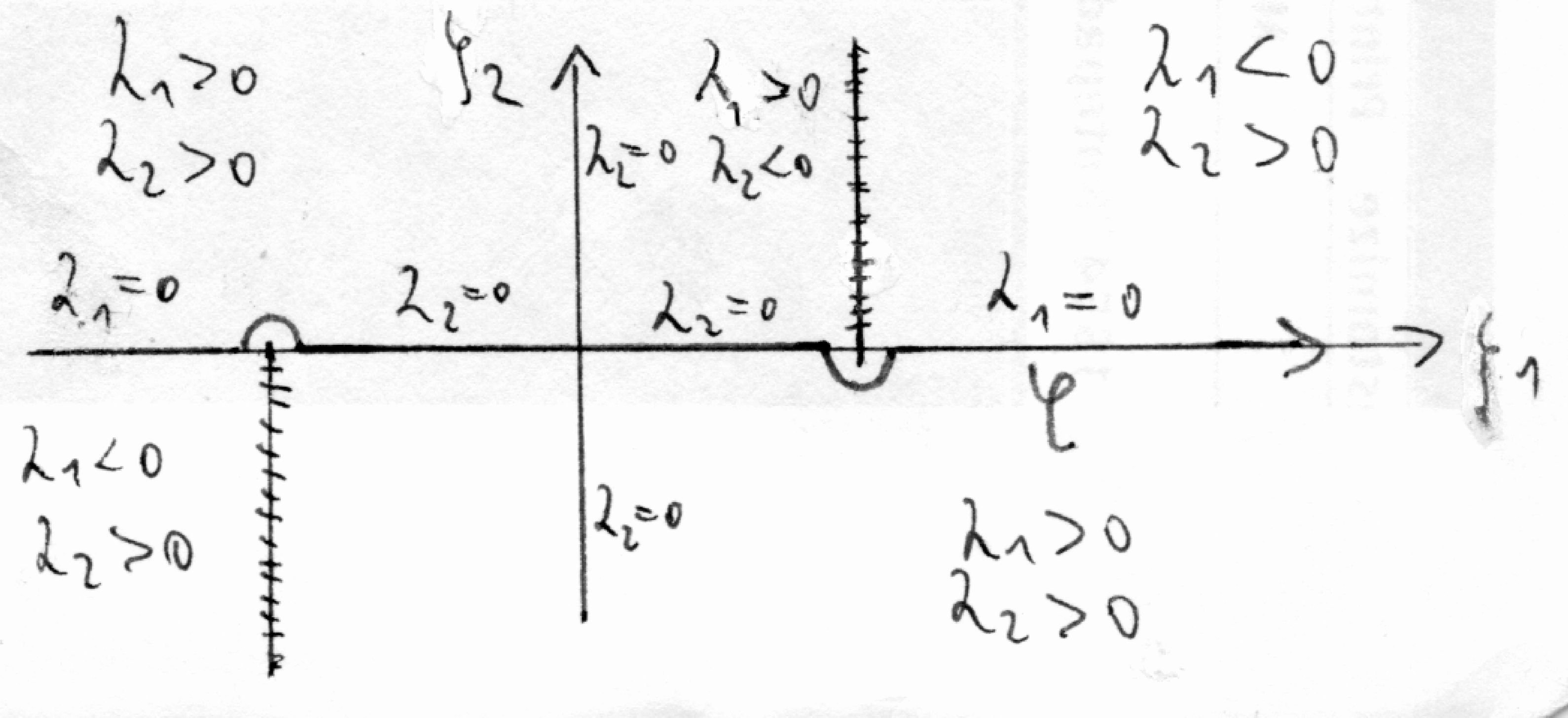
$$\boxed{G(r,\varphi,z;r',\varphi',z') = \frac{i}{8\pi}\sum_{m=-\infty}^{\infty} e^{im(\varphi-\varphi')}\int_{-\infty\,\mathcal{C}}^{\infty} d\zeta\, e^{i\zeta(z-z')}\, J_m(\lambda r_<)\, H_m^{(1)}(\lambda r_>)} =$$

$$= \frac{i}{8\pi}\sum_{m=-\infty}^{\infty}(2-\delta_{m0})\cos[m(\varphi-\varphi')] \int_{\mathcal{C}} d\zeta\, e^{i\zeta(z-z')}\, J_m(\lambda r_<)\, H_m^{(1)}(\lambda r_>)$$



$$\lambda(\zeta) = \lambda(-\zeta)$$

$k_2 = 0, \quad k = k_1$



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