3.8.1.2 Potenz $z^{\frac{p}{n}}$; $n, p \in \mathbb{N}$; n, p fix vorgegeben.

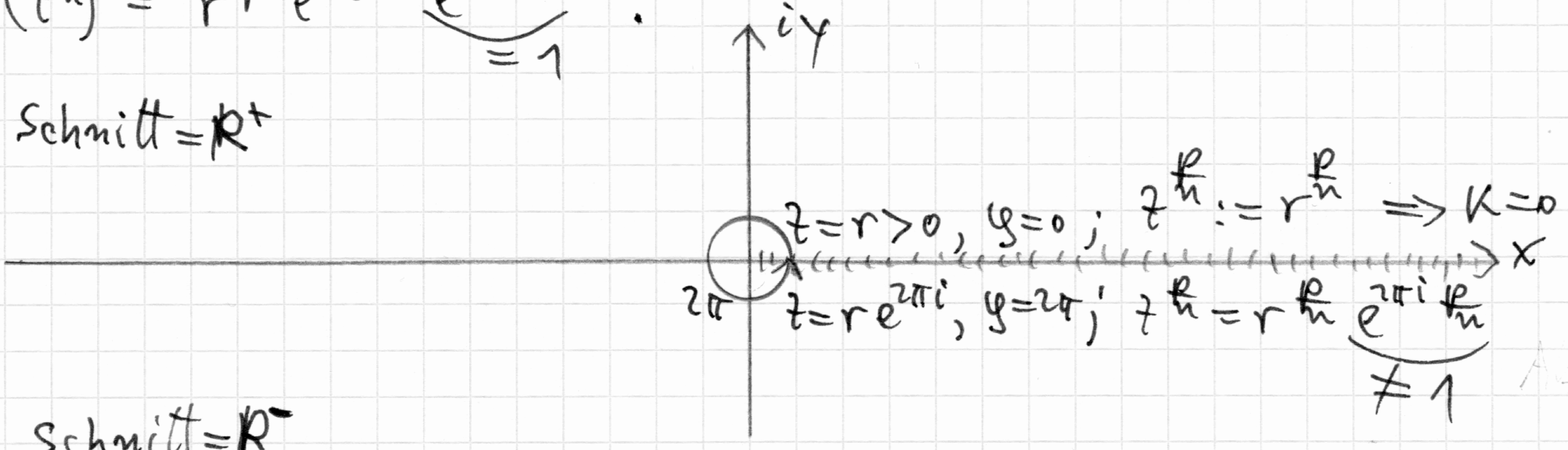
Verzweigungspunkte: $z = 0, \infty$

$z = re^{i\varphi}$

$z^{\frac{p}{n}} = r^{\frac{p}{n}} e^{i\varphi\frac{p}{n}} e^{2\pi i \frac{p}{n} k}$, $k = 1, 2, \dots, n (\equiv 0)$, n Zweige

$(z^{\frac{p}{n}})^n = r^p e^{i\varphi p} \underbrace{e^{2\pi i p k}}_{=1}$.

Schnitt $= \mathbb{R}^+$



Schnitt $= \mathbb{R}^-$

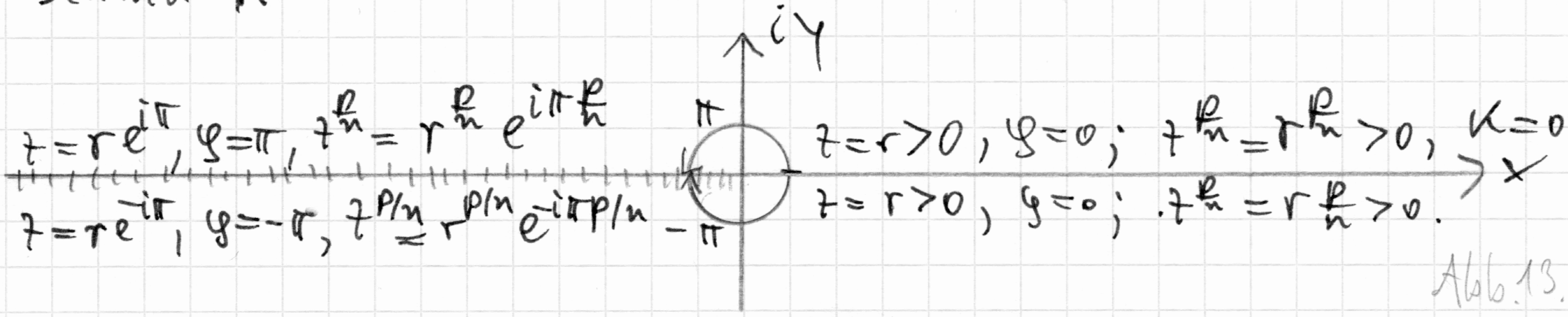
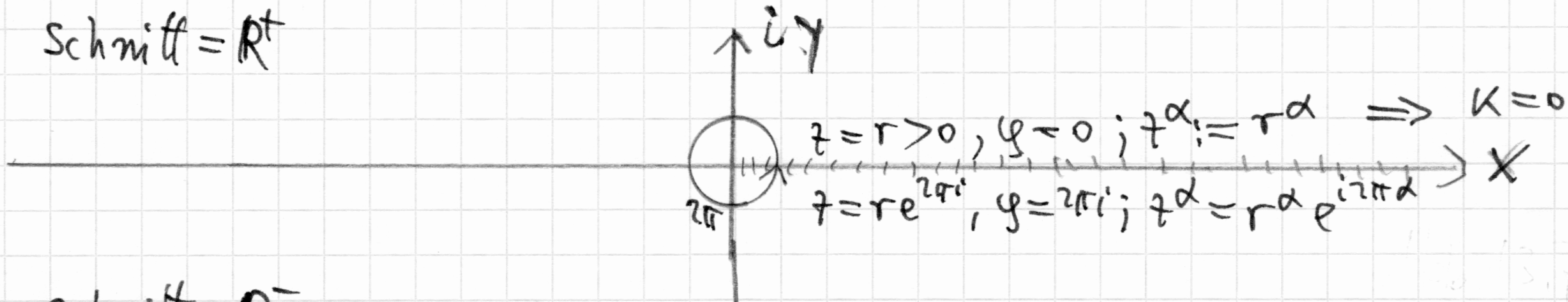


Abb. 13.5

3.8.1.3 Potenz z^α, $\alpha \in \mathbb{R} \setminus \mathbb{Q}$, ∞ viele Zweige

$z = re^{i\varphi}$, $z^\alpha = r^\alpha e^{i\varphi\alpha} e^{i2\pi\alpha k}$, $k \in \mathbb{Z}$

Schnitt $= \mathbb{R}^+$



Schnitt $= \mathbb{R}^-$

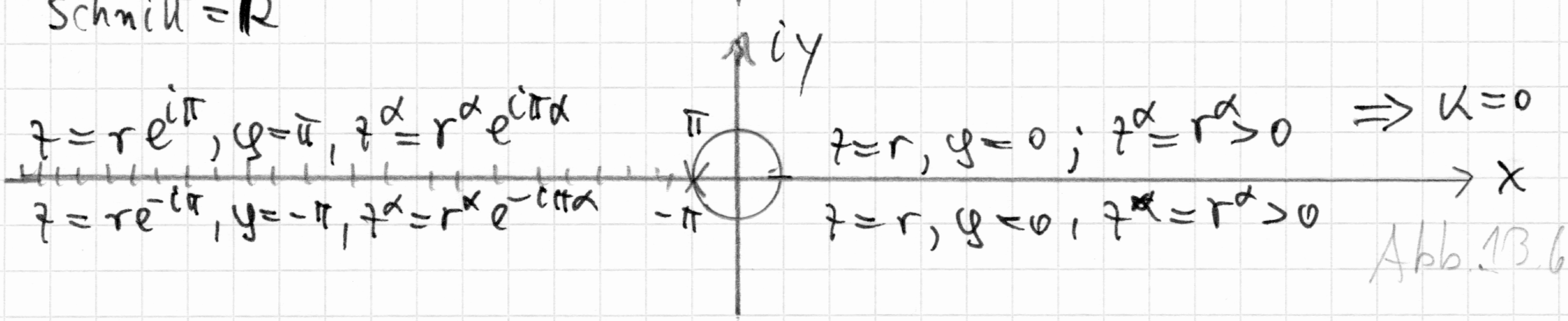


Abb. 13.6

Berechnung des Integrals:

$$J = \int_0^\infty \frac{x^{\beta-1}}{1+x^2}\, dx = \frac{\pi}{2 \sin\frac{\pi\beta}{2}}, \quad 0 < \beta < 2.$$

$$f(z) = \frac{z^{\beta-1}}{1+z^2}$$