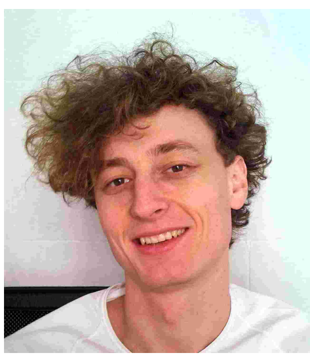
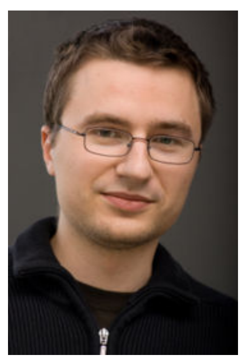
Bound states and non-equilibrium time evolution in 1d strongly interacting lattice models

Hans Gerd Evertz, TU Graz



Martin Ganahl



Elias Rabel

Fabian Essler

Masud Haque

and R. Vlijm, D. Fioretto, M. Brockmann, J.-S. Caux

Outline

- Propagation of bound states in the XXZ chain
  - Ferromagnet
  - Antiferromagnet at finite magnetization
  - Nonintegrable models
- Scattering of bound states
  - XXZ
  - Bose-Hubbard
  - Hubbard

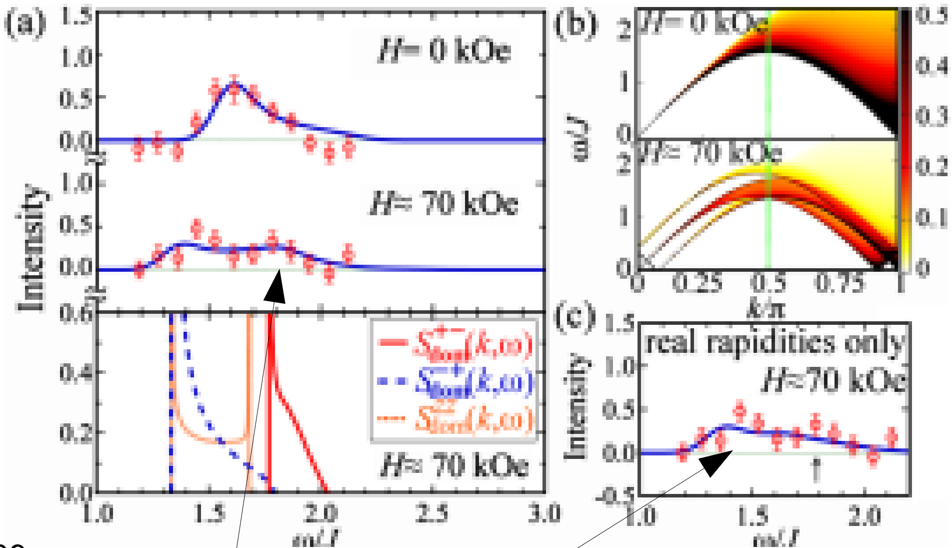
XXZ Heisenberg spin ½ chain

$$H_{XXZ} = \sum_i \frac{J_{xy}}{2} \left(S_i^+ S_{i+1}^- + S_{i+1}^- S_i^+ \right) + J_z S_i^z S_{i+1}^z \,, \qquad \Delta = \frac{J_z}{J_{xy}}$$

$$H_{sf} = \sum_i t \left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i \right) + V(n_i - \frac{1}{2})(n_{i+1} - \frac{1}{2}) \,, \qquad \frac{V}{t} = 2\Delta$$

- There are bound states ("M-strings")
- Difficult to see in standard condensed matter experiments

Caux et al J Stat.Mech 2005
Pereira, White, Affleck PRL 2008, PRB 2009
Sashi et al, PRB 2011



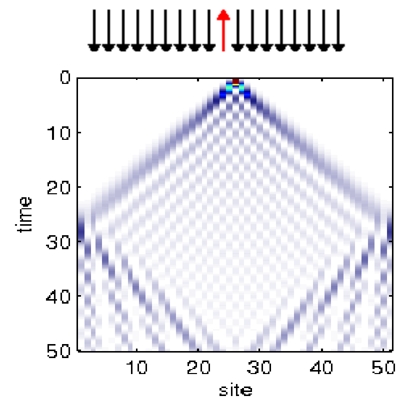
Kohno PRL 2009
$CuCl_2 \cdot 2N(C_5D_5)$

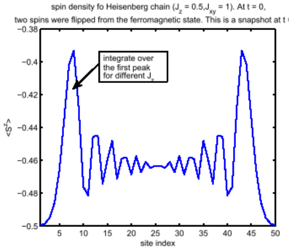
Spectra with and without bound state contributions

- Here: study with Local Quantum Quench (ED, tebd, Bethe)

Single particle excitation: magnon

- Initial state: FM groundstate (empty lattice), with local quench at center site (inf. magn. field)





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- Same as a single fermion (=> time evolution)

$$|\psi(t=0)\rangle \;=\; c^\dagger_{x=0}|0\rangle \;=\; \sum_{k} c^\dagger_{k}|0\rangle$$

- Dispersion is -J_X cos k , thus velocities J_X sin k

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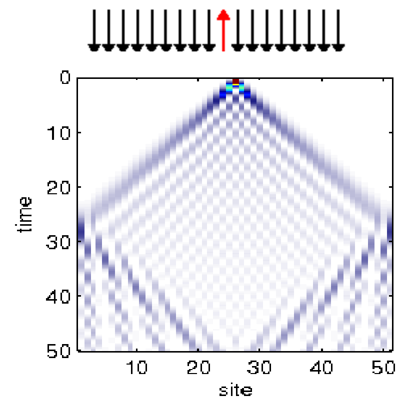

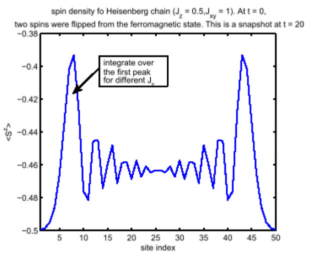

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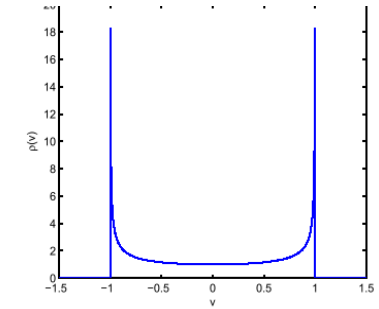
- Dispersion is $-J_X$ cos k , thus velocities J_X sin k

- many k-modes, around π/2, with almost maximum velocity Jx

↔ Lieb Robinson bound   Lieb,Robinson Comm.Math.Phys 1972
Sims,Nachtergaele arXiv:1102.0835



Single particle excitation: magnon

- Initial state: FM groundstate (empty lattice),
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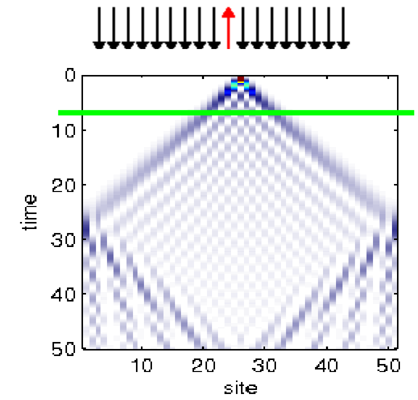

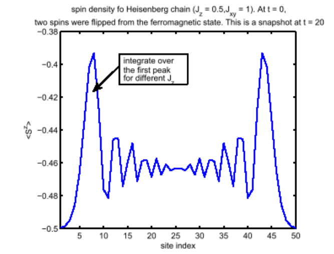

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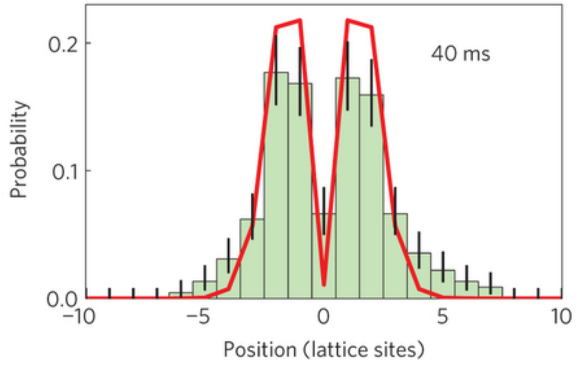
- Same as a single fermion (=> time evolution)

$$|\psi(t=0)\rangle \;=\; c^\dagger_{x=0}|0\rangle \;=\; \sum_k c^\dagger_k|0\rangle$$

- Recent cold atom lattice experiment

  Fukuhara et al. (Munich) Nature Physics 9, 235 (2013)

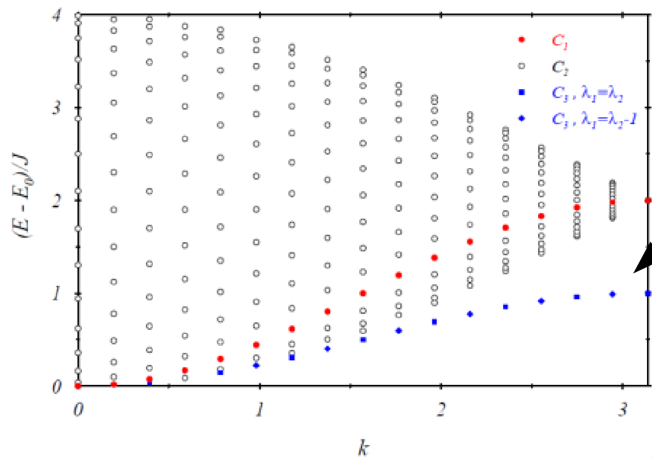


Bound states

- Bethe ansatz: $$|\psi\rangle_{L-r} = \sum_{1\le n_1<\ldots<n_r\le L} \sum_{\mathcal{P}} exp\left(i \sum_{j=1}^{r} k_{\mathcal{P}_j} n_j + \frac{i}{2} \sum_{l<j} \Theta_{\mathcal{P}_l \mathcal{P}_j} \right) \quad |n_1 \ldots n_r\rangle$$

n_i : position of particle number i
Total of r particles

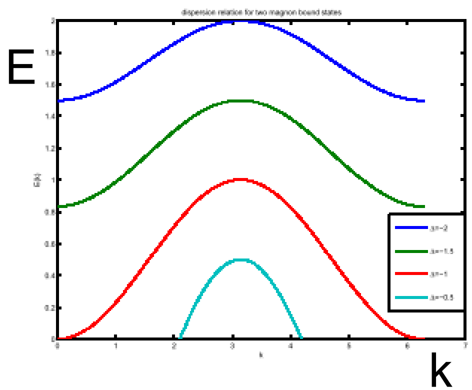
- Two-magnon excitation spectrum:
  (Karbach,Müller '97)



Bound state (2-string)

- Dispersion relation of M-string:

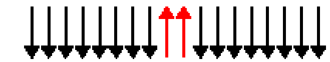
$$E = \frac{\sin\nu}{\sin M\nu} \underbrace{[\cos M\nu - \cos k]}_{>0}, \qquad J_z = \cos\nu$$



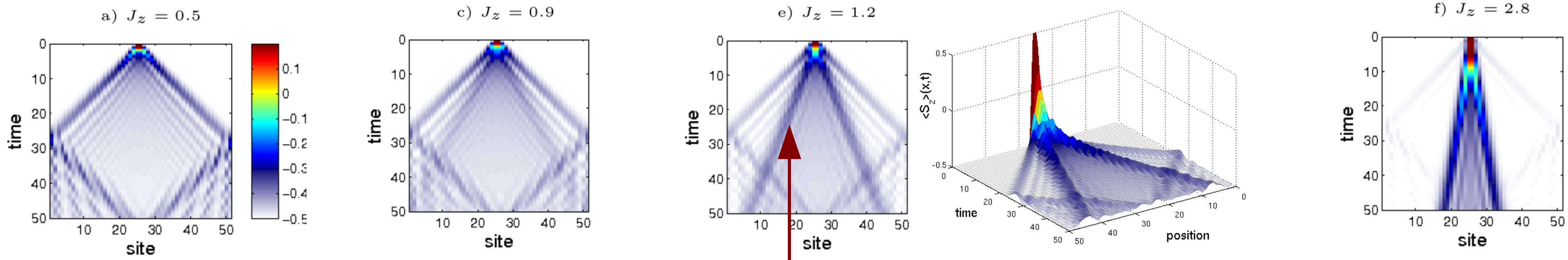
- Requires $J_z > \cos\frac{\pi}{M}$

- Momentum constrained; $k = \frac{\pi}{2}$ with max. velocity $\frac{\sin\nu}{\sin M\nu}$ present when $J_z > \cos\frac{\pi}{2M}$

Two-spin excitation in FM

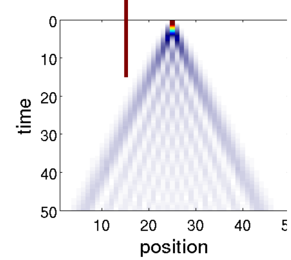


S_z



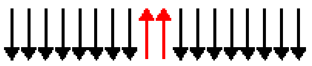
$J_z = 0.5$ $J_z = 0.9$ $J_z = 1.2$ $J_z = 2.8$

- Two distinct branches beyond $J_z = 0.7$
- New lower branch is bound state

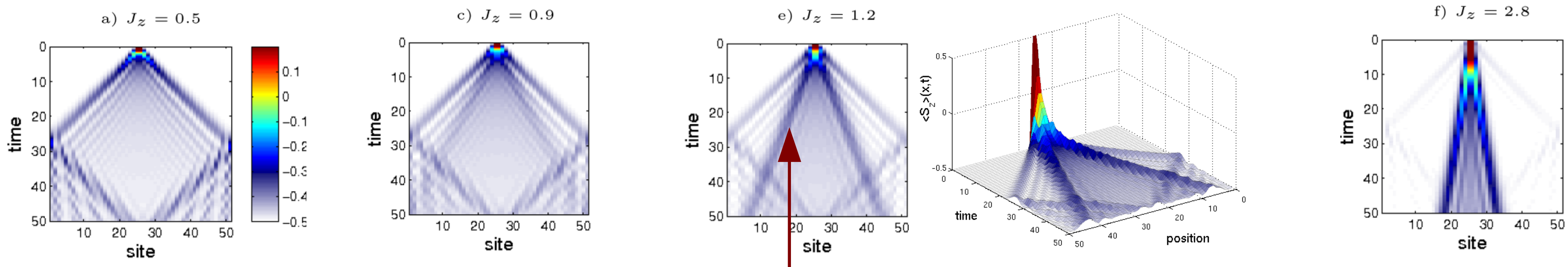


$P(\uparrow\uparrow)$

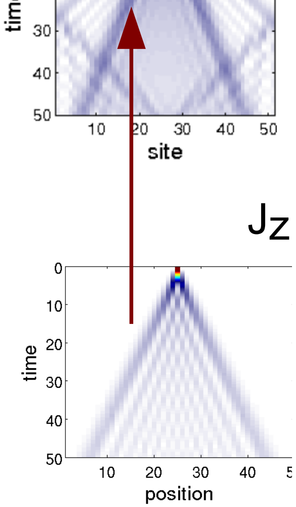
Two-spin excitation in FM



S_z



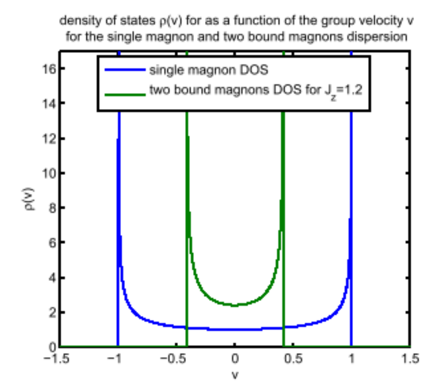
$J_z = 0.5$ $J_z = 0.9$ $J_z = 1.2$ $J_z = 2.8$



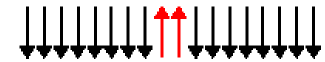
P(↑↑)

- Two distinct branches beyond $J_z = 0.7$
- New lower branch is bound state
- Bethe: 2-string: linear dispersion appears at $J_z > \frac{1}{\sqrt{2}}$

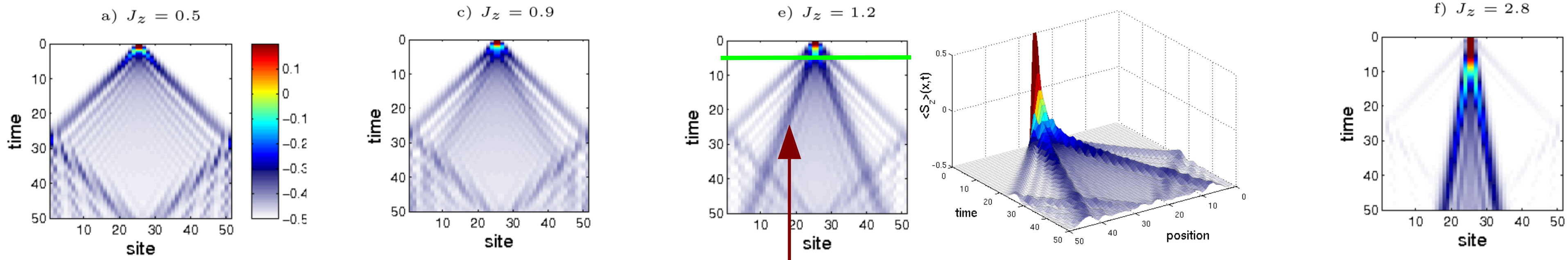
Maximum velocity = $\frac{1}{2J_z}$



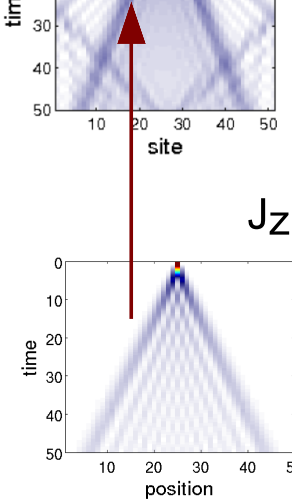
Two-spin excitation in FM



S_z

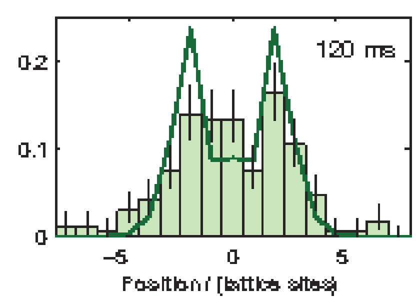


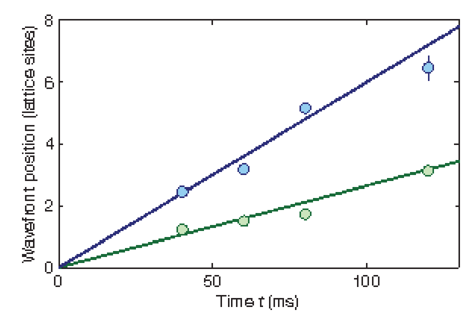
$J_Z = 0.5$ $J_Z = 0.9$ $J_Z = 1.2$ $J_Z = 2.8$



P(↑↑)

- Two distinct branches beyond $J_Z = 0.7$
- New lower branch is bound state
- Observed in cold atom experiment
  (following our proposal)
  Fukuhara et al. (Munich) Nature 502, 76 (2013)

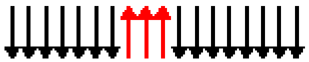




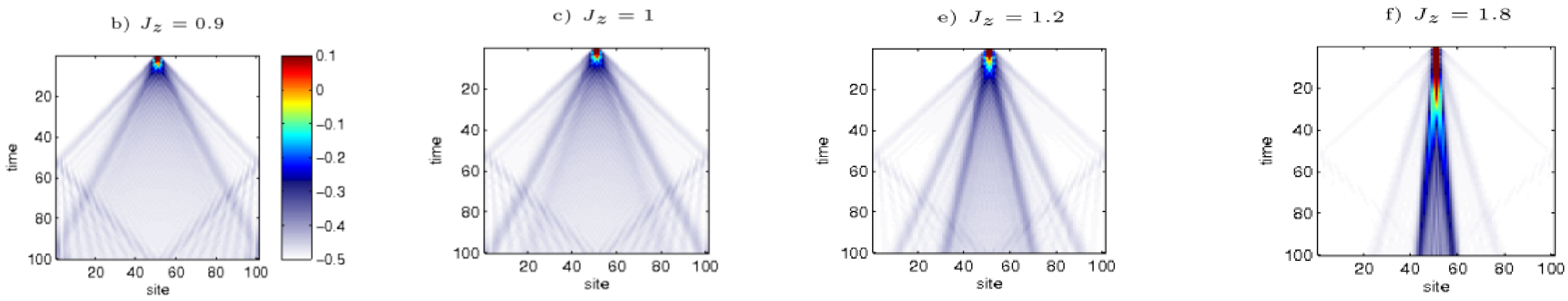
- Note: the sign of H and J_Z does not matter for time evolution *from a given initial state !*

U. Schneider et al., Nature Physics 8, 213 (2012) (supplement, for Hubbard model)

Bound states of 3 spins

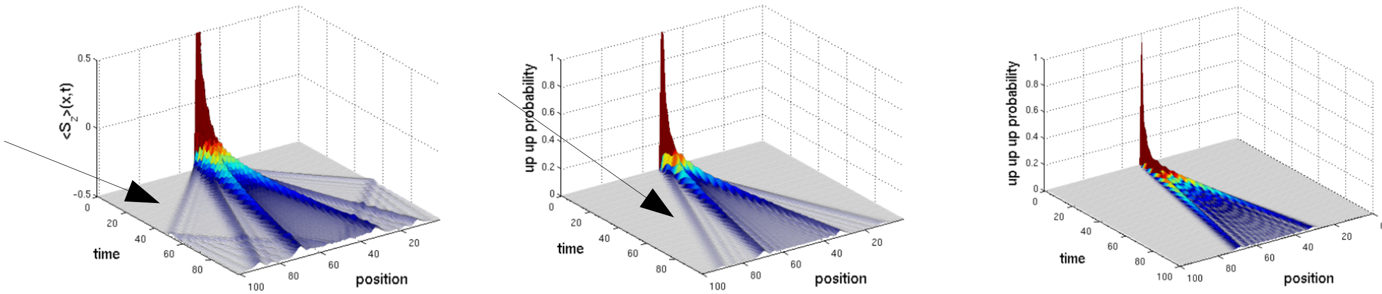


S_z



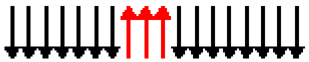
$J_Z = 0.9$ $J_Z = 1.0$ $J_Z = 1.2$ $J_Z = 1.8$

- Three propagating branches, of 1, 2, and 3 particles:

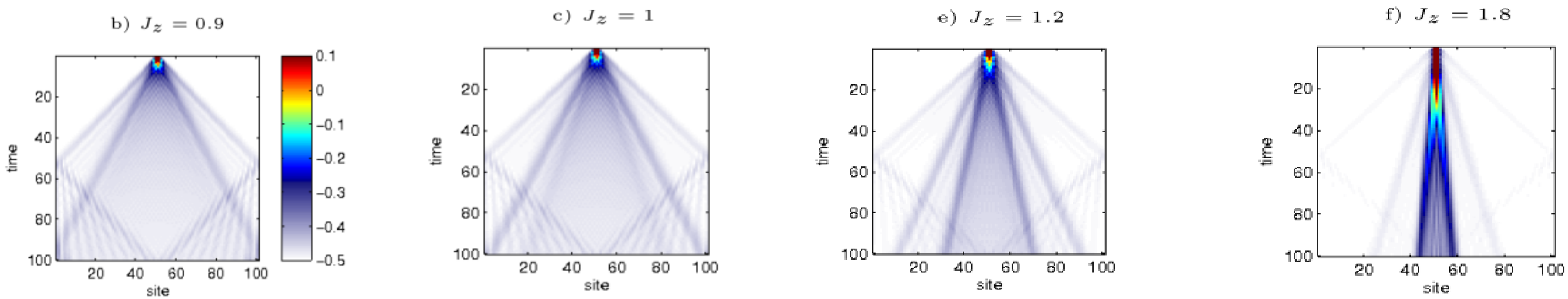


$J_Z = 1.2$: S_Z P(↑↑) P(↑↑↑)

Bound states of 3 spins



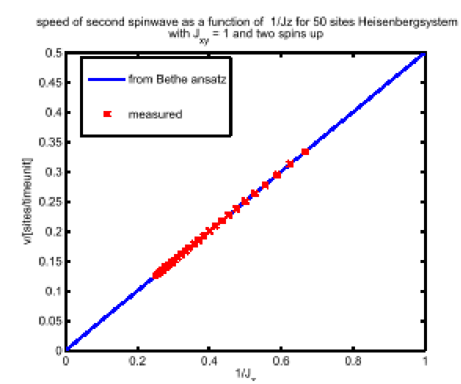
S_z

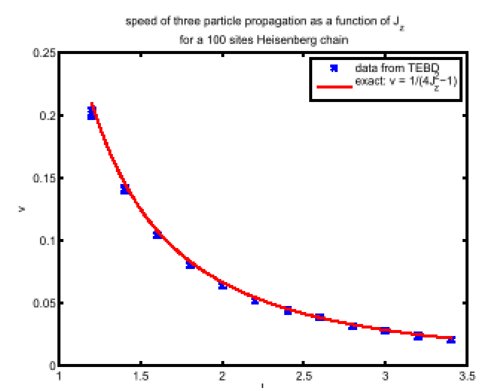


$J_Z = 0.9$ $J_Z = 1.0$ $J_Z = 1.2$ $J_Z = 1.8$

- Velocities of branches agree with Bethe ansatz $$v_{max} = \frac{\sin \nu}{\sin M\nu}$$

(M=2, M=3)



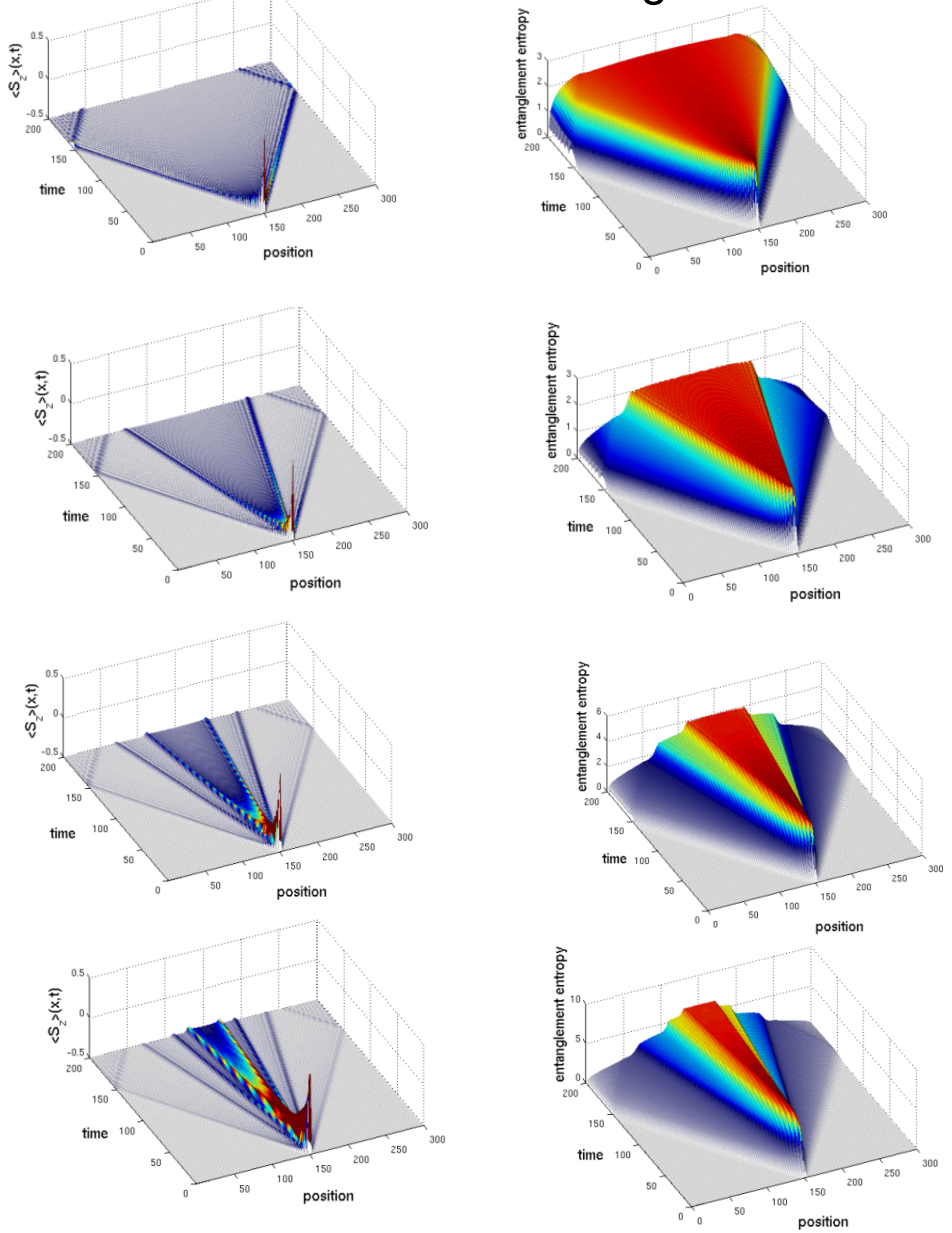


Bipartite Entanglement (x,t) between Left and Right of site x

- 2 particles, $J_Z = 0.5$
  (no bound state)
- 2 particles, $J_Z = 1.2$:

  Entanglement *saturates*,
  with a *step structure*
- 3 particles
- 4 particles

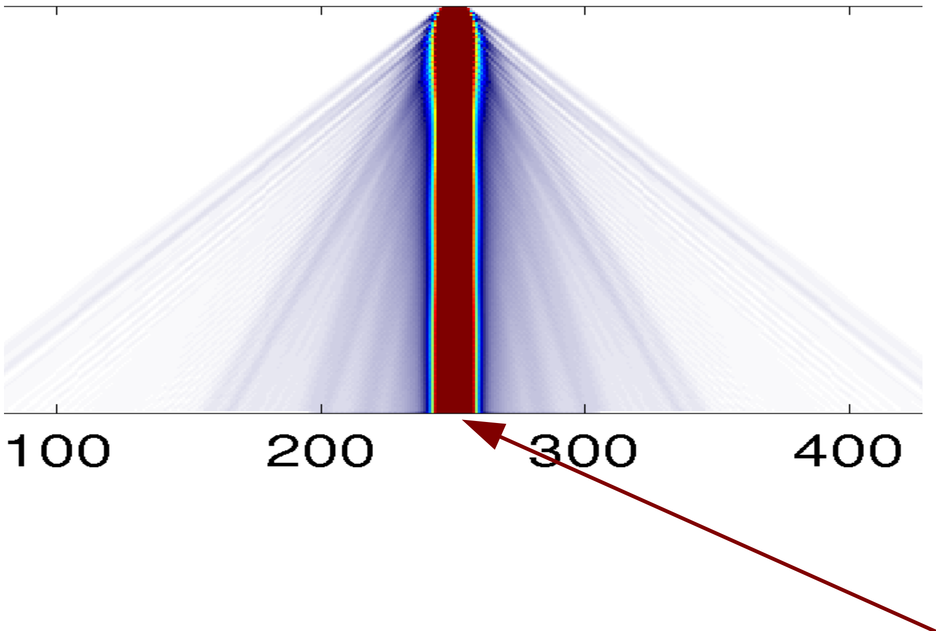
S_z entanglement



Initial block of 10 spins at $J_Z = 1.1$

- Block of spins is not an eigenstate, decays into substrings

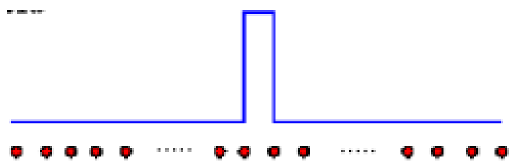
  ("evaporative cooling")



- Eigenstates have exponentially decaying spatial wave function (wide at $J_Z = 1.1$)

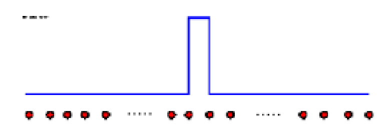
Local quench in the **AF** groundstate at non-zero magnetization

- Prepare ground state with a local infinite magnetic field, then switch field off

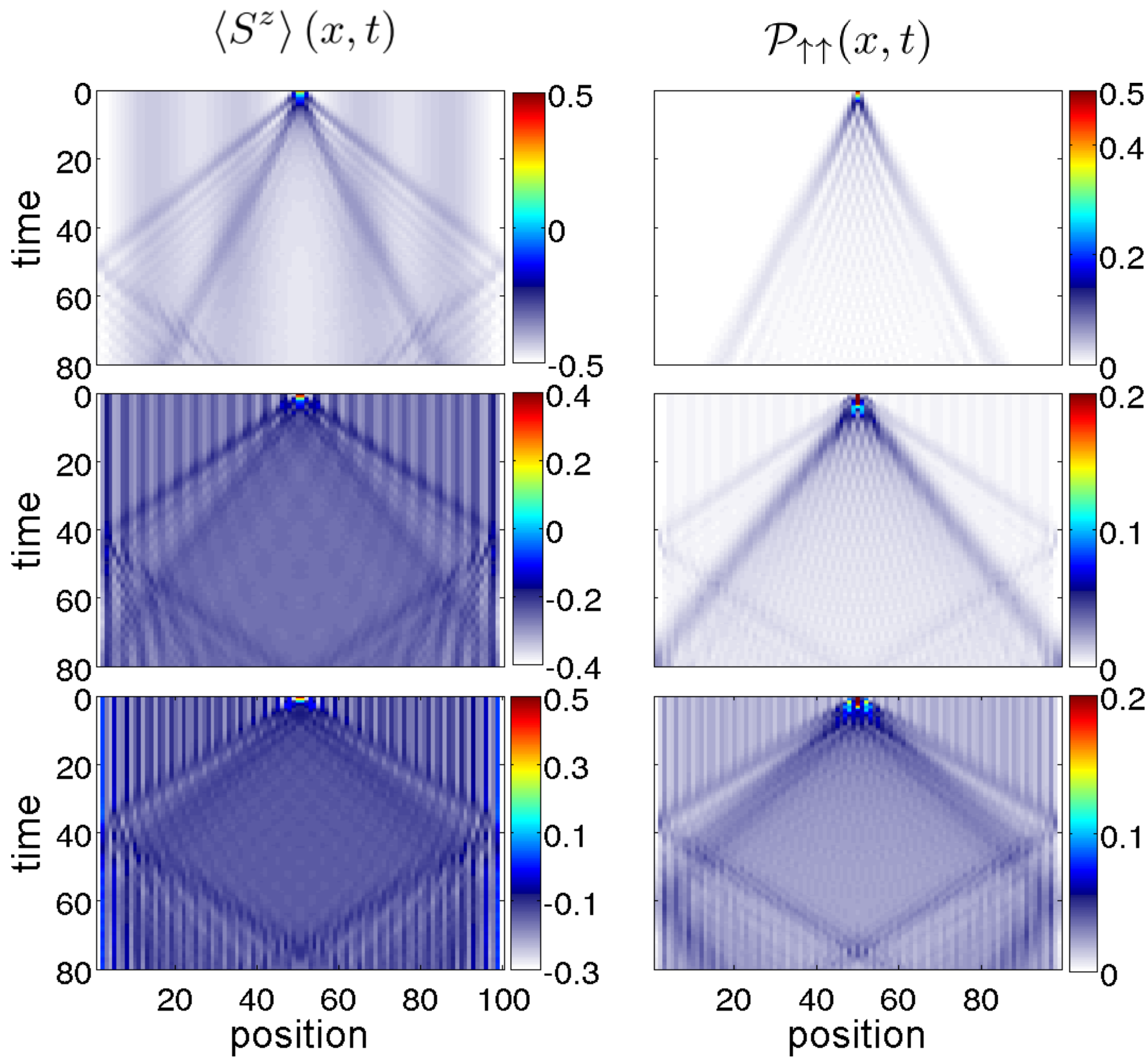


- AF at nonzero magnetization is in the Luttinger liquid phase *for any J_z*
- Highly entangled ground state. Spinon excitations.
- Do bound "string-states" remain visible ?
- Accessible in cold atom experiments

Evolution from AF groundstate at J_z=1.2, finite magnetization, 2 spins fixed up

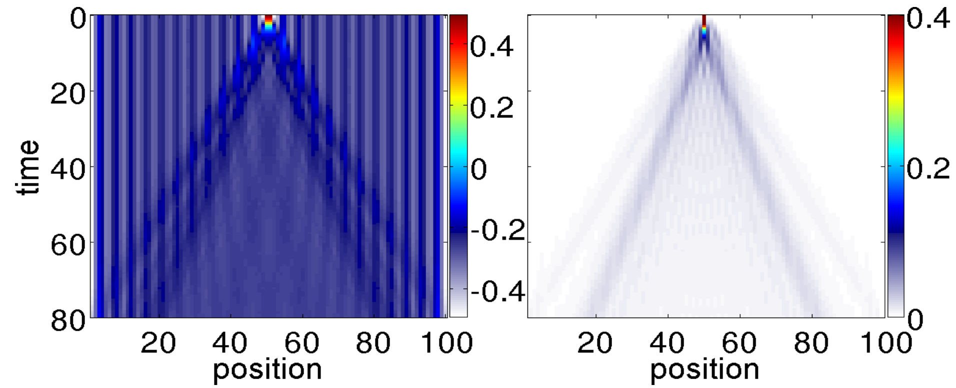


- Low filling 6% (=large magnetization): like magnons and bound magnons
- Larger filling 24% Larger velocity
- Filling 36%: fewer momenta contribute to bound state → washed out

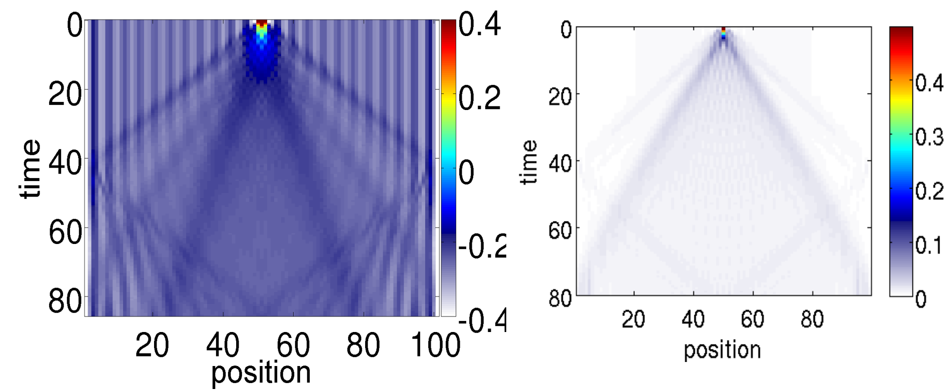


Non-integrable models

- Experiments may not precisely reproduce the XXZ model
- Bound states remain visible
- Next-nearest neighbor coupling J/10



- Chain in parabolic field (“optical trap”)

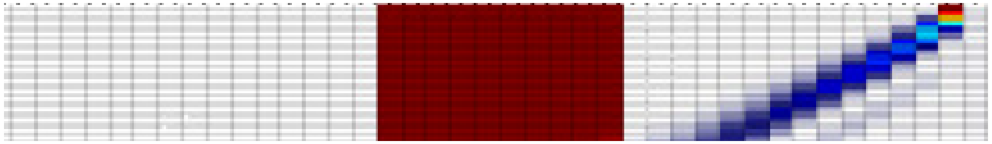


Scattering of bound states

(or: What do Bethe phase shifts do ?)

Scattering of magnon and bound state

- Magnon hits a “stable” wall of bound particles (almost string eigenstate)



$$\Delta = 10 \quad \left(v \sim \frac{1}{\Delta^{M-1}} \right)$$

Scattering of magnon and bound state

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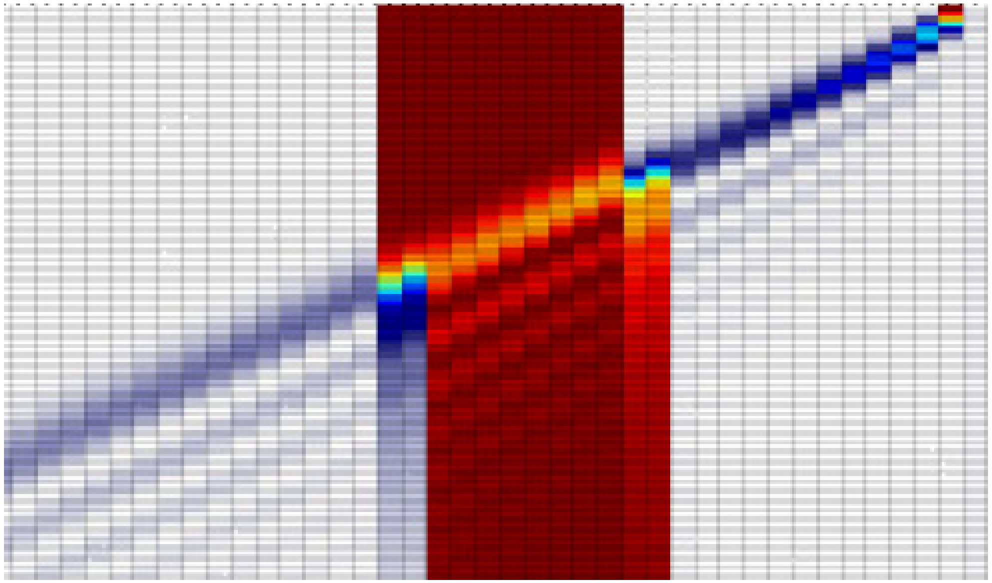
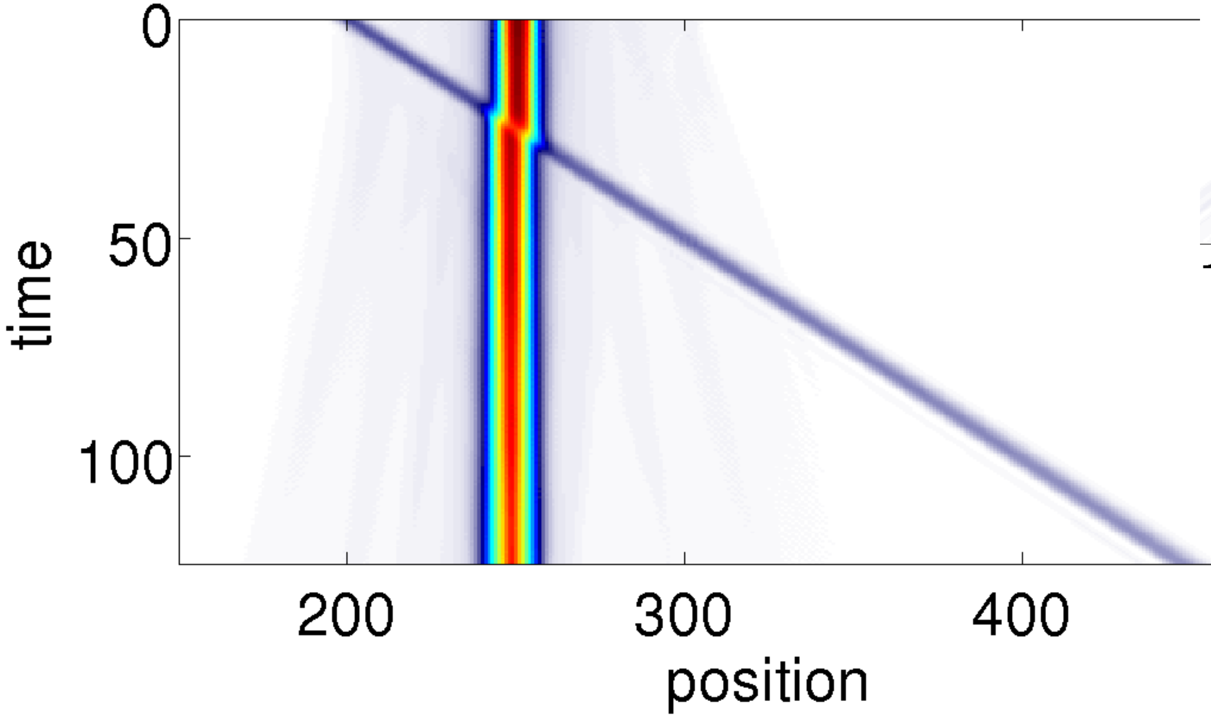


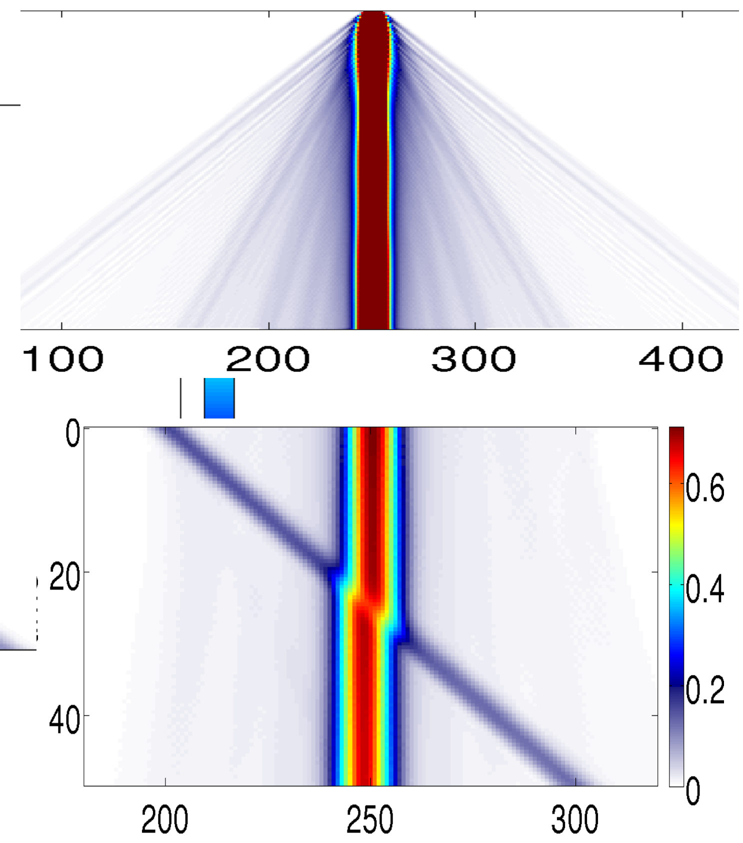
$$\Delta = 10 \quad \left(v \sim \frac{1}{\Delta^{M-1}} \right)$$

- Integrable model: no diffraction, no backward scattering
- A *hole* moves through the wall
- Resembles one pass of *Newtons Cradle,* but wall moves by ***two*** lattice sites

Not an effect of large couplings

- Phenomena remain the same at small coupling: Here $\Delta = 1.1$

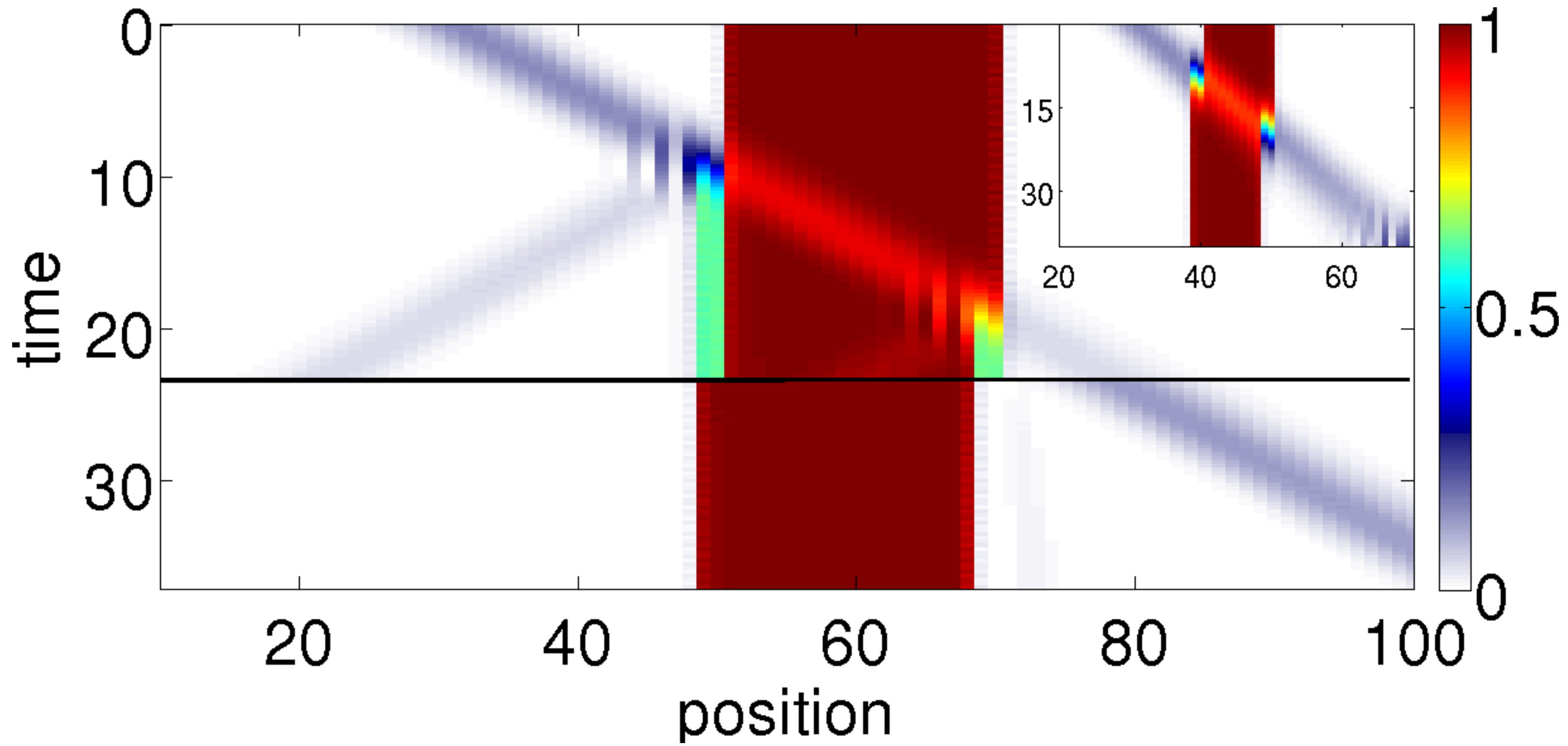




- Wall stabilized before scattering by evaporative cooling
- At small Δ, the M-particle eigenstate (wall) is much wider than M sites
- Incoming Gaussian superposition of magnons exits wall apparently unchanged

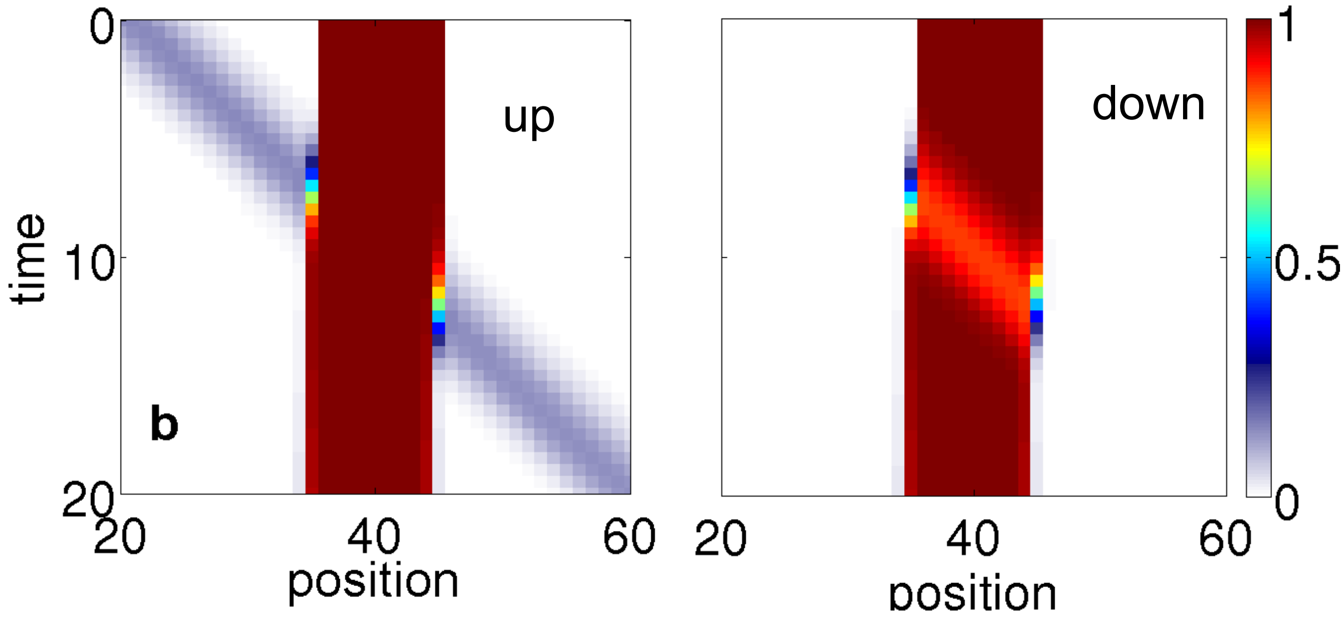
Role of integrability

- XXZ with nnn coupling: non-integrable: backscattering
- Inset: different nnn coupling, integrable: no backscattering



Fermi Hubbard model

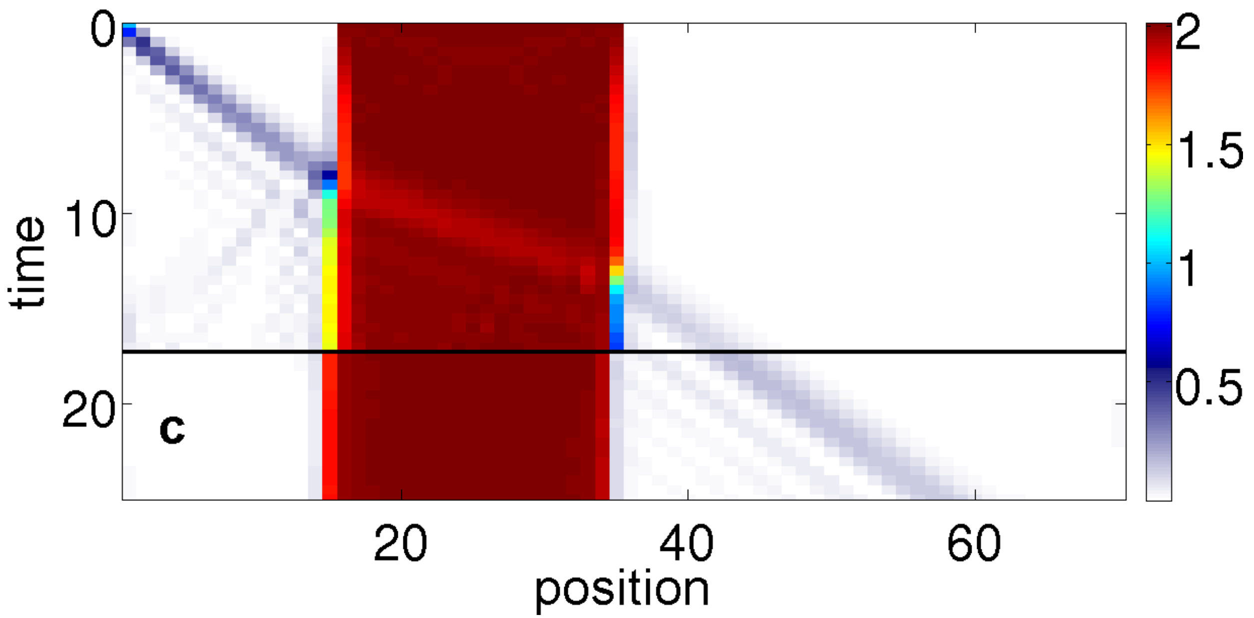
- Wall of doubly occupied sites, U=100



- Integrable: no backscattering. Particle-hole transmutation
- Incoming up-spin particle is transmitted as a down-spin hole
- Wall moves by one doubly-occupied site

Bose Hubbard model

- Wall of doubly occupied sites, U=30, incoming single magnon

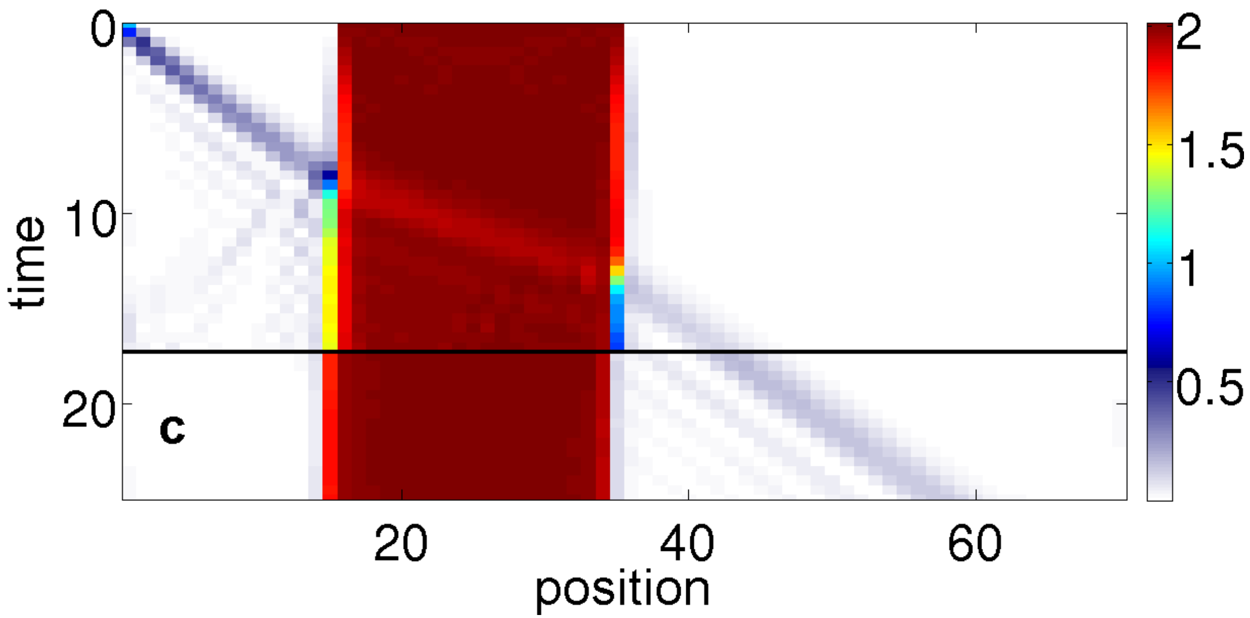


- Not integrable: partial reflection, partial particle-hole transmutation

.

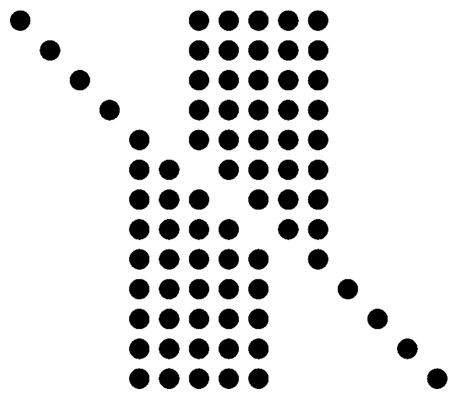
Bose Hubbard model

- Wall of doubly occupied sites, U=30, incoming single magnon



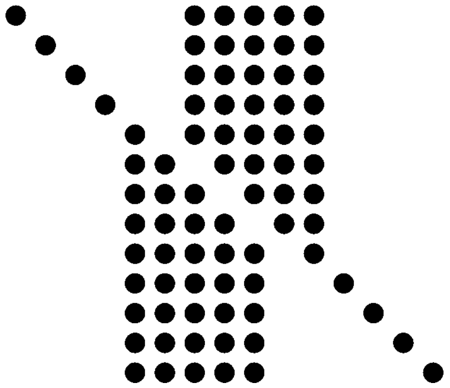
- Not integrable: partial reflection, partial particle-hole transmutation
- Bottom part: **projection** onto cases in which a particle is present on the right
- Then the complete wall moves by one doubly-occupied site
- Effects also visible at smaller U

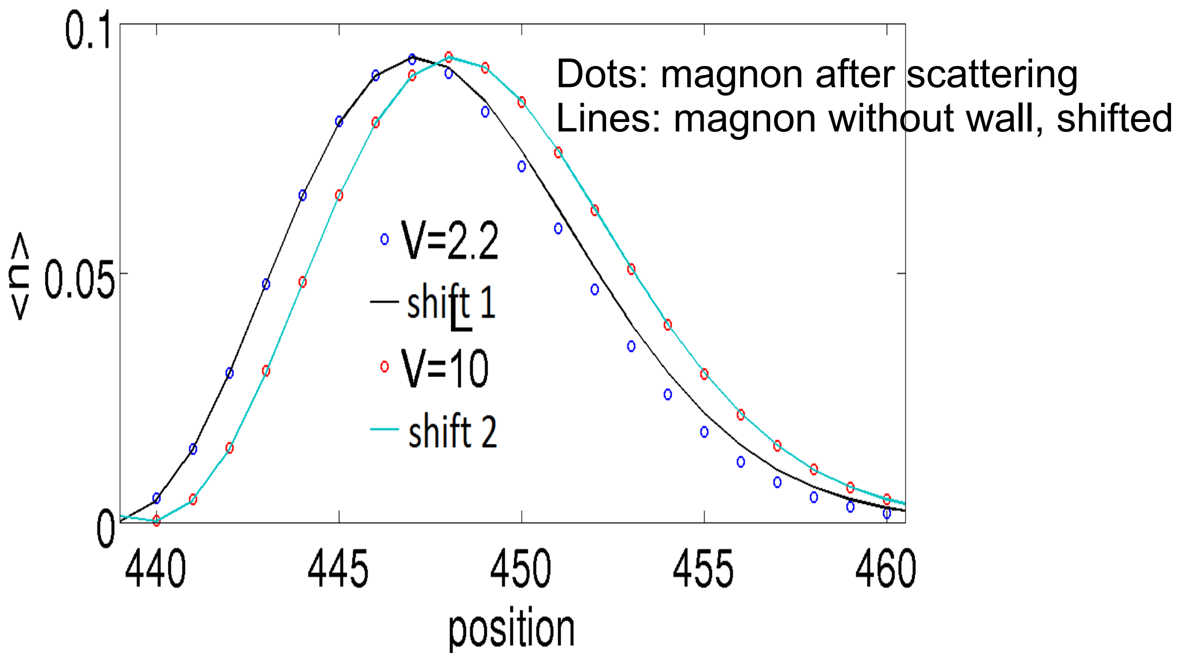
Semiclassical picture (large coupling)



- Incoming particle cannot touch wall because of energy conservation
- Energy current has to continue
- A particle from inside the wall has to move left → hole propagates
- Picture implies that transmitted particle should jump forward by 2 sites !

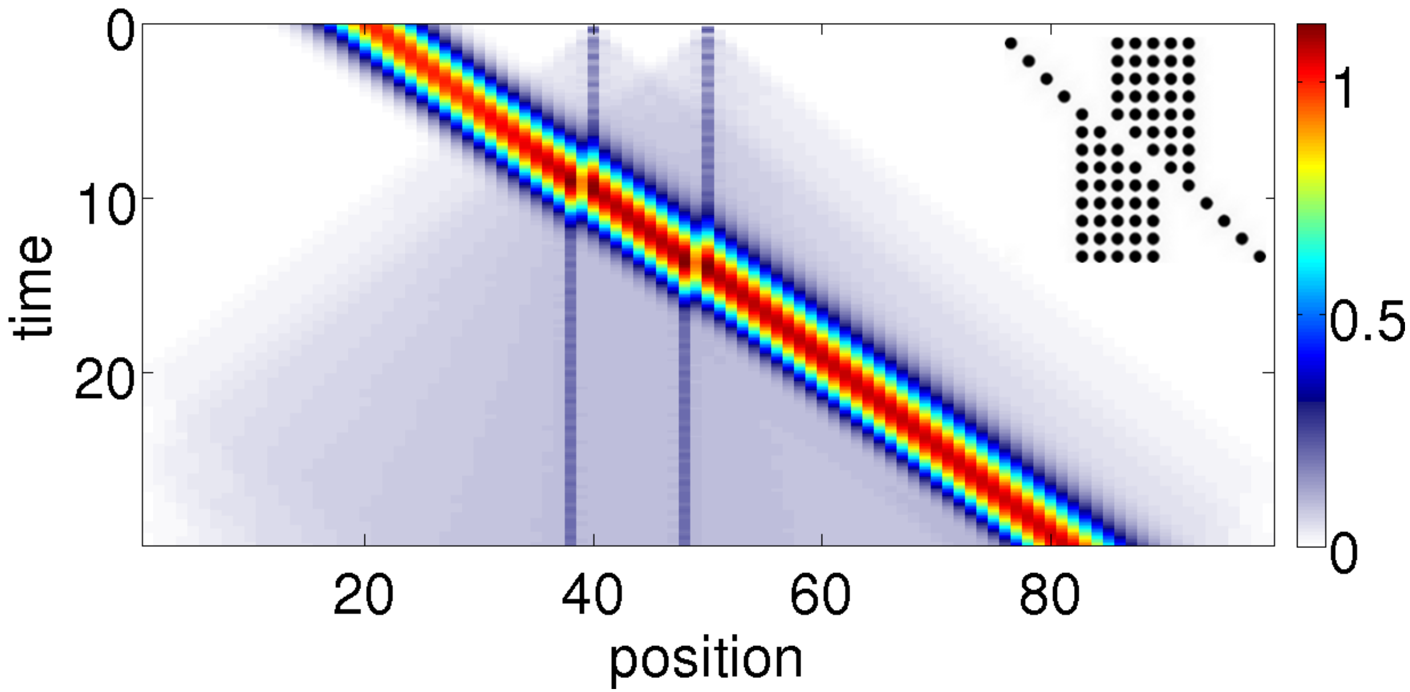
Semiclassical picture





- Incoming particle cannot touch wall because of energy conservation
- Energy current has to continue
- A particle from inside the wall has to move left → hole propagates
- Picture implies that transmitted particle should jump forward by 2 sites
- At large V, an incoming Gaussian is indeed transmitted unchanged, with shift 2 (i.e. momentum-independent phase shift)

Bipartite entanglement entropy



- Incoming Gaussian is entangled internally
- Jumps visible
- Almost no additional entanglement between wall and outgoing particle: Product state, no diffraction

Scattering phase shifts from Bethe ansatz

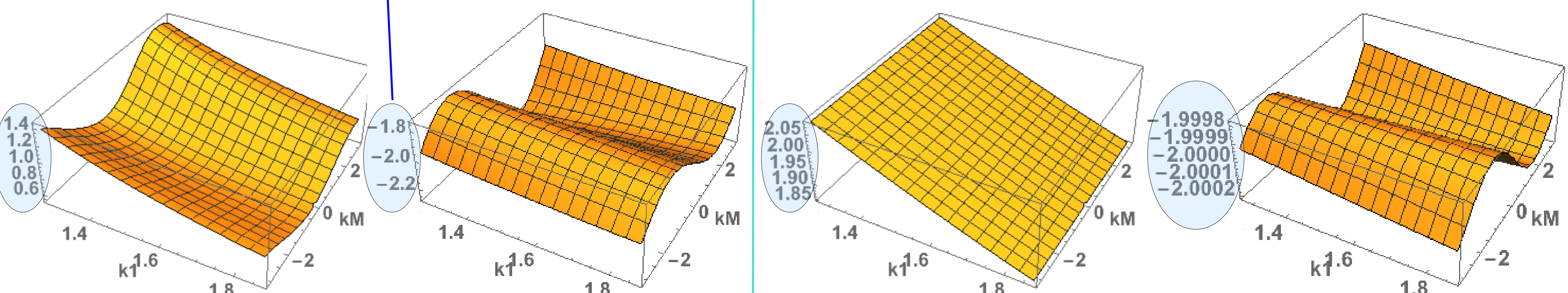
$$\Theta_{nm}(x) \equiv \begin{cases} \theta_{|n-m|}(x) + 2\theta_{|n-m|+2}(x) + \ldots + 2\theta_{n+m-2}(x) + \theta_{n+m}(x) \\ \quad \text{for} \quad n \neq m, \\ 2\theta_2(x) + 2\theta_4(x) + \ldots + 2\theta_{2n-2}(x) + \theta_{2n}(x) \quad \text{for} \quad n = m. \end{cases}$$

$$\theta_n(x) = 2\tan^{-1}\left(\frac{\tan\frac{x\phi}{2}}{\tanh\frac{n\phi}{2}}\right) + 2\pi\left[\frac{\phi x + \pi}{2\pi}\right]$$

$$x = \alpha_n - \alpha_m$$

(The quantity "x" of the Bethe ansatz is like a momentum!)

- Slope of Theta → displacement
- Example: Displacements vs momenta (Magnon scattered by M=5 string):



Magnon Wall Magnon Wall

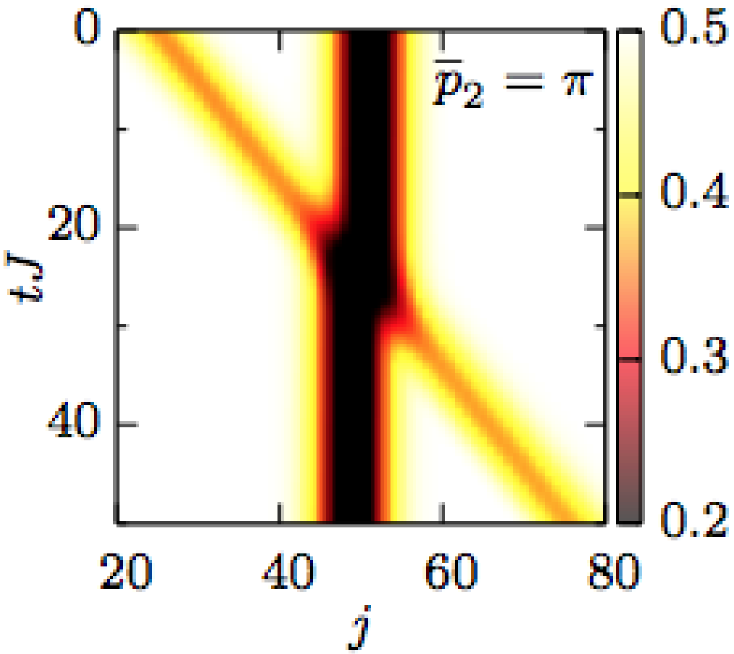
Delta = 1.1 Delta = 5

Scattering of String Eigenstates, Bethe ansatz

R Vlijm, M. Ganahl, D. Fioretto, M. Brockmann, M. Haque, HGE, J.-S. Caux, arxiv:1507.08624

= Phys. Rev. B 92, 214427 (2015)

- Start from eigenstates (instead of sets of strings)
- Prepare Gaussian superpositions around desired momenta and locations
- Exact time evolution



Δ=2, 1-string on 3-string

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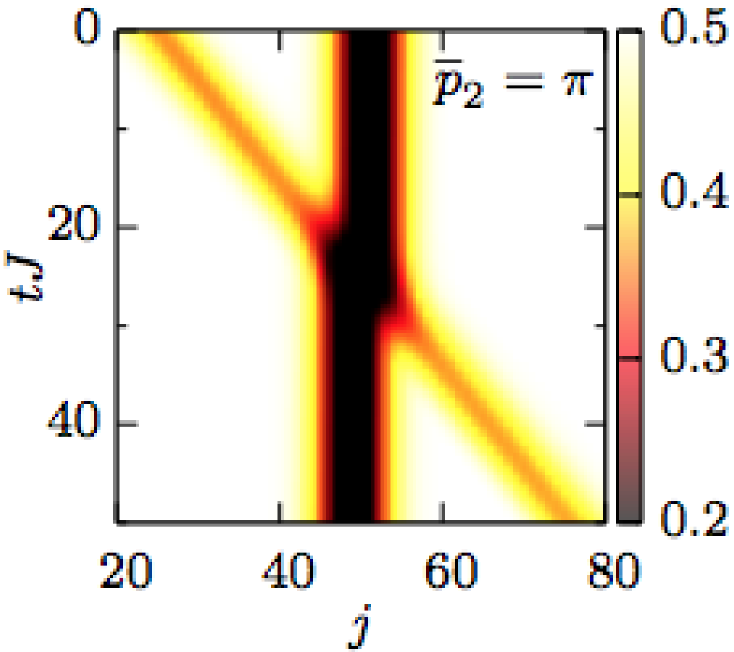


Δ=2, 1-string on 3-string

Limits of displacements (analytical):

At large width M: (scatter 1-string off M-string)

Displacement = $2 + O(e^{-(M-1)\text{acosh}\Delta})$

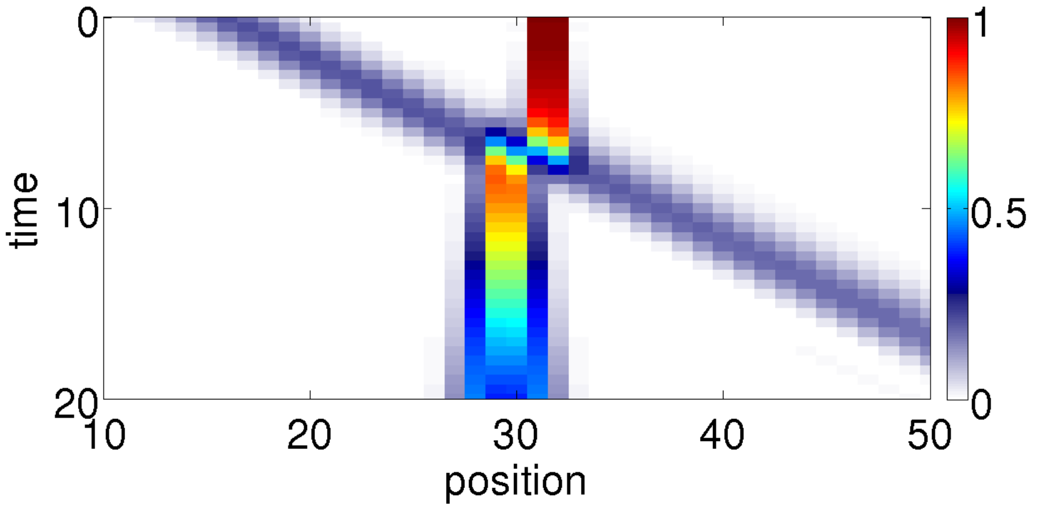
At large Δ: (scatter N-string off M-string)

Displacement = $2\min(N, M) - \delta_{NM}$

Different initial states

How many sites ?

- Wall of 2 sites is enough



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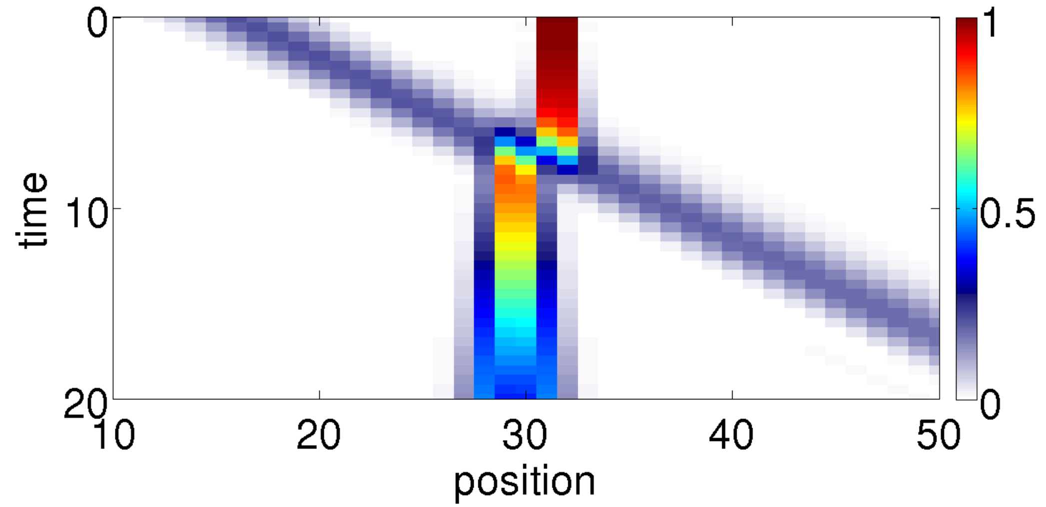
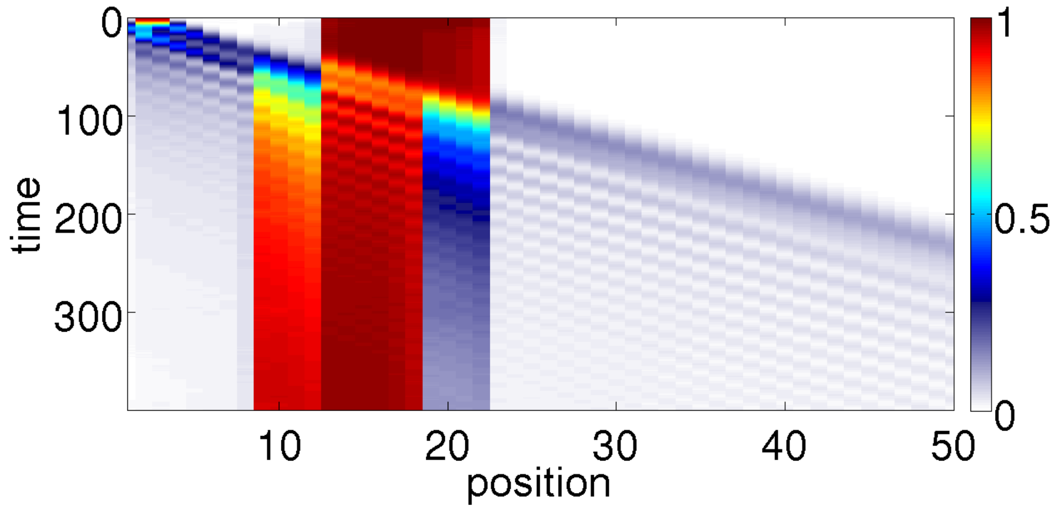
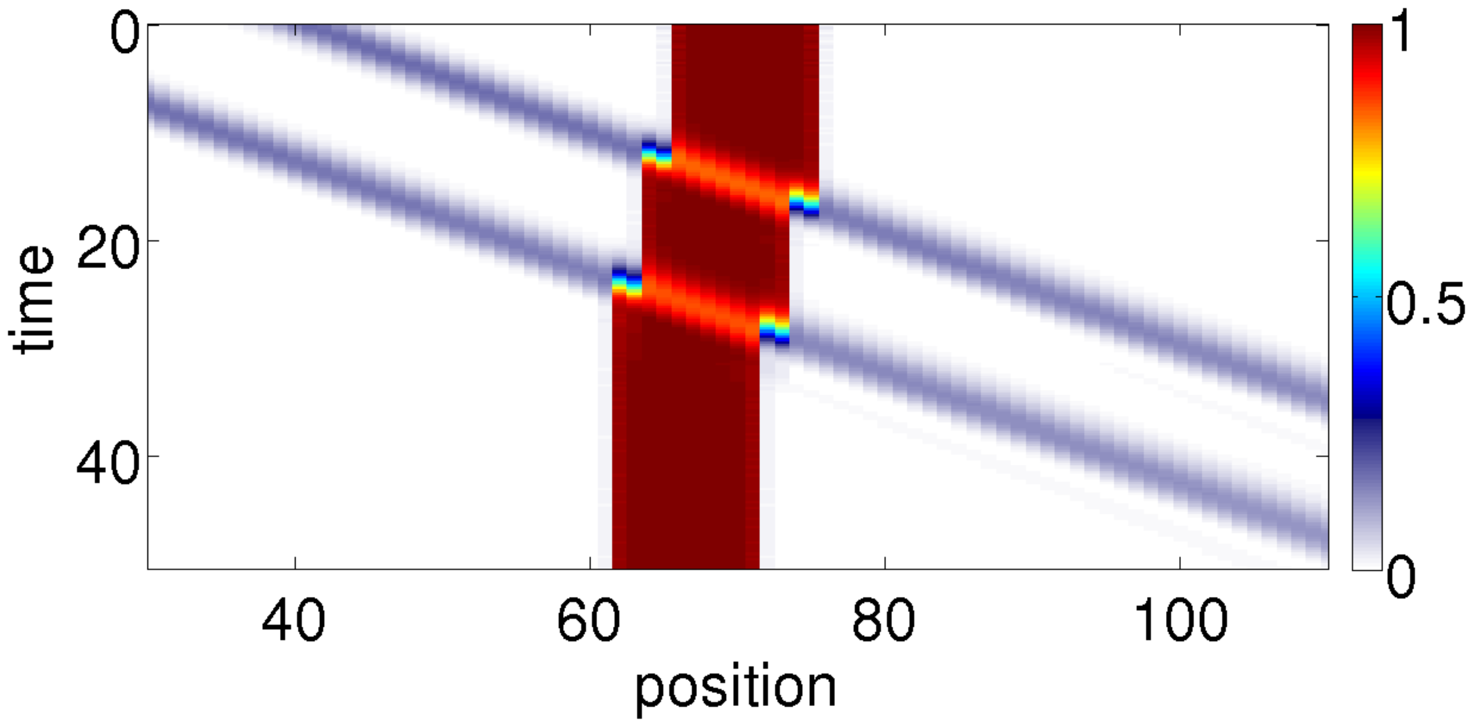


- Incoming two-magnon state. Wall shifts by 4 sites.



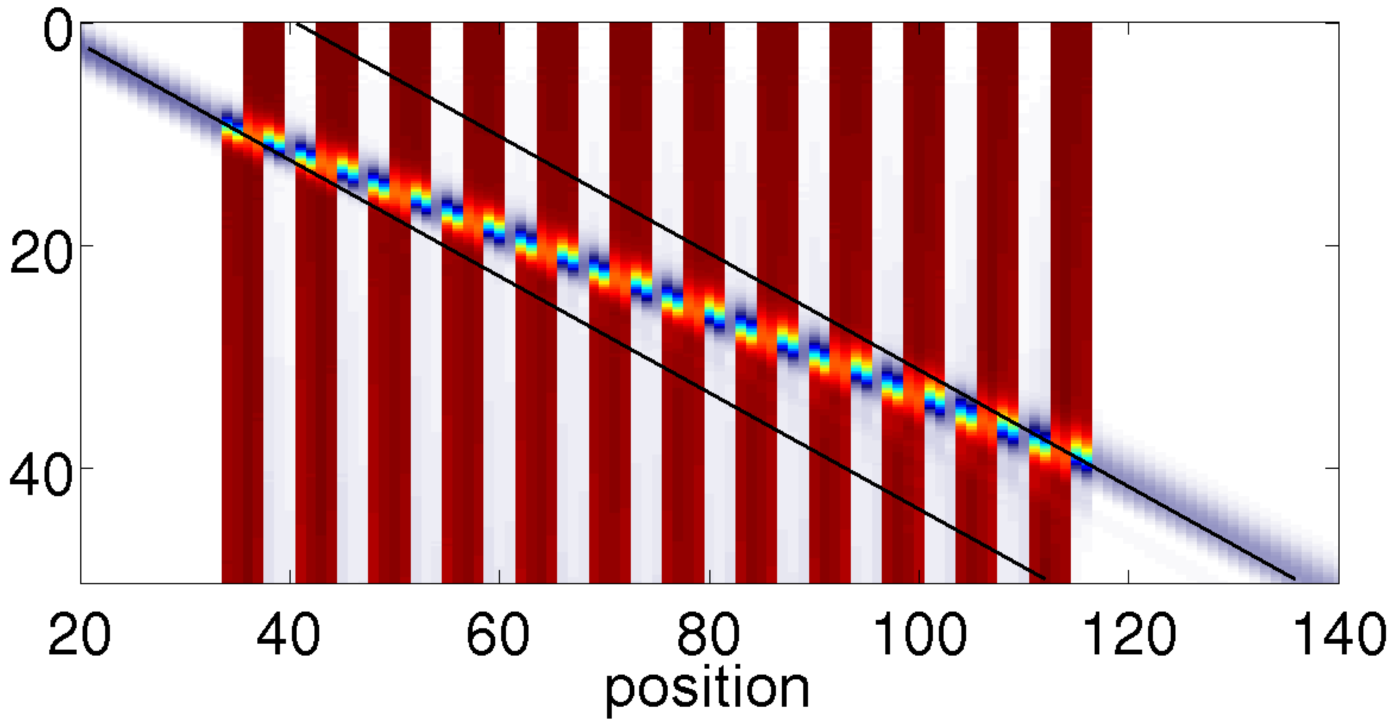
"Shift register"

- Shifts wall coherently; counts passing particles



Metamaterial with “supersonic” mode

- Set up a superlattice of many walls

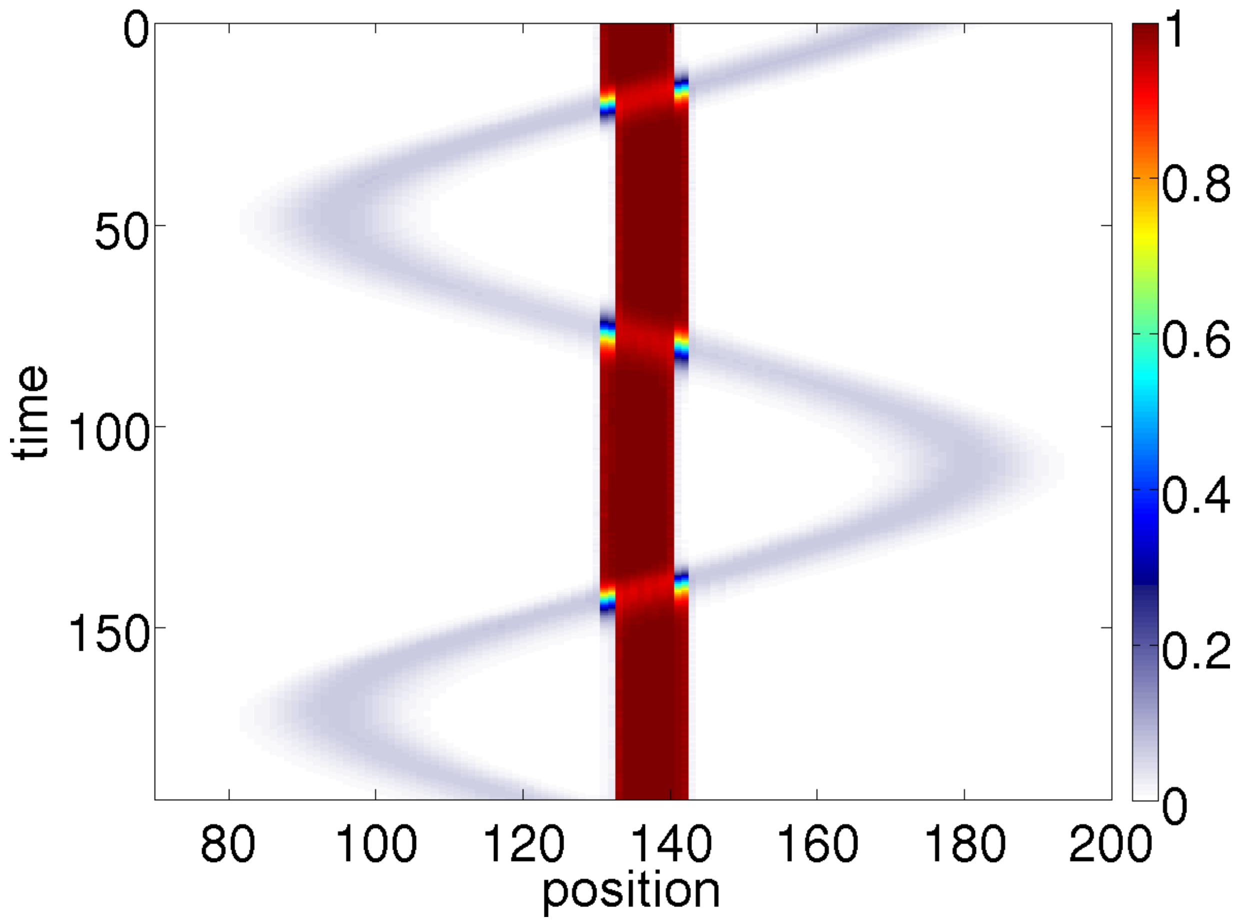


- At each wall, a passing particle jumps forward by 2 sites

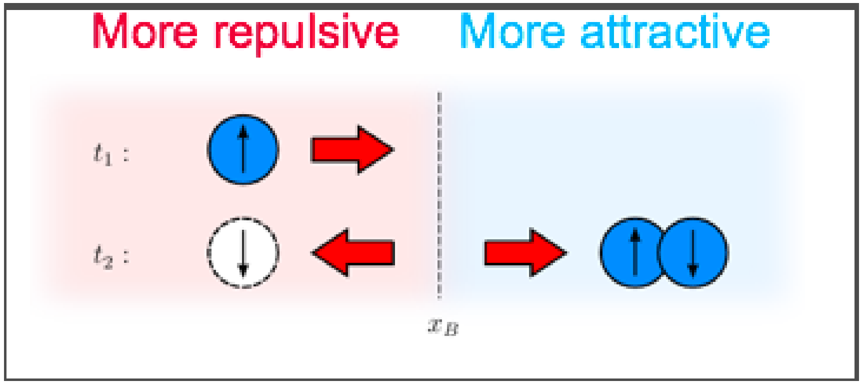
→ Average velocity larger than on empty lattice

Lattice Quantum Newton's Cradle

- Place system into a field → Bloch oscillations



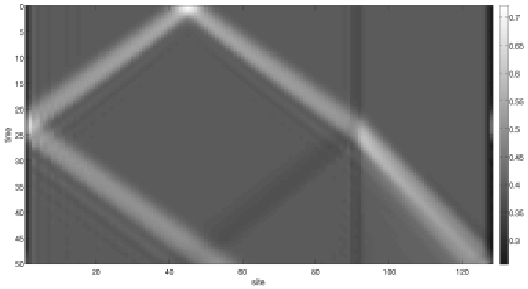
Andreev-like reflection



- Left and Right regions, with different couplings

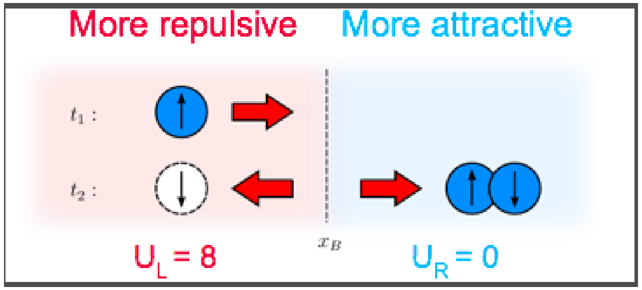
- Luttinger liquid, small excitation: Andreev-like reflection when $\gamma = \frac{K_L - K_R}{K_L + K_R}$ is negative
  i.e. **when right side is more attractive (or less repulsive) than left**
  (Safi & Schulz 1996, hydrodynamic approximation)

- Simplest case: spinless fermions (no pairing)
  V_L=0, V_R= -1
  (cf. Daley et al, 2008)

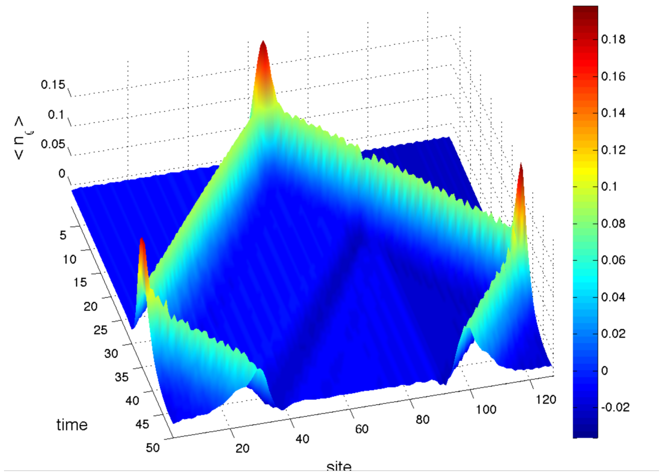


Andreev-like reflection

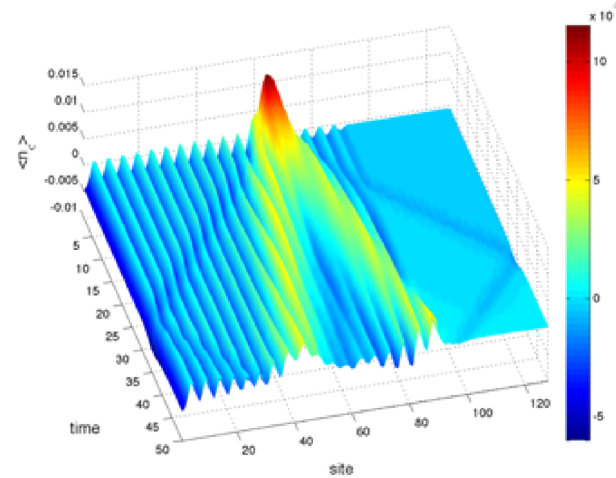
Hubbard chain (quarter filling, $U_L = 8$, $U_R = 0$)



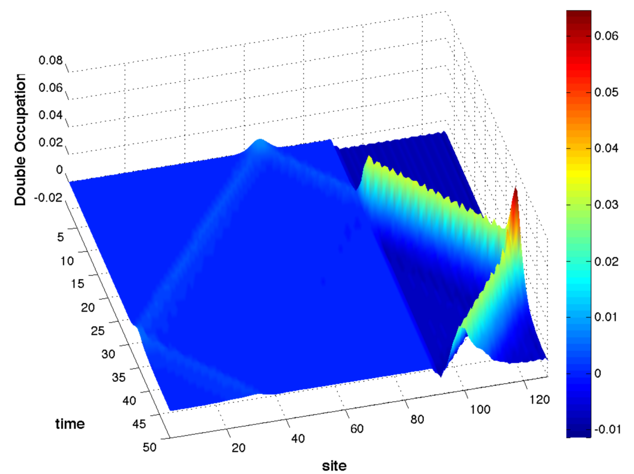
Charge:



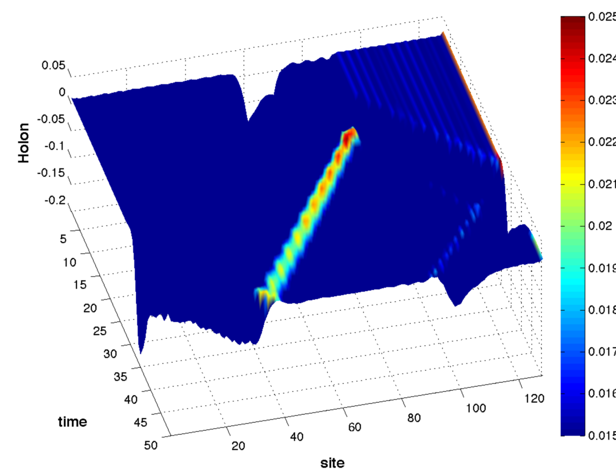
Spin:
(eventual normal reflection)



Double occupation:



Holes:



- Reflection coefficient agrees with prediction
- **Also for repulsive → less repulsive, or free → attractive**

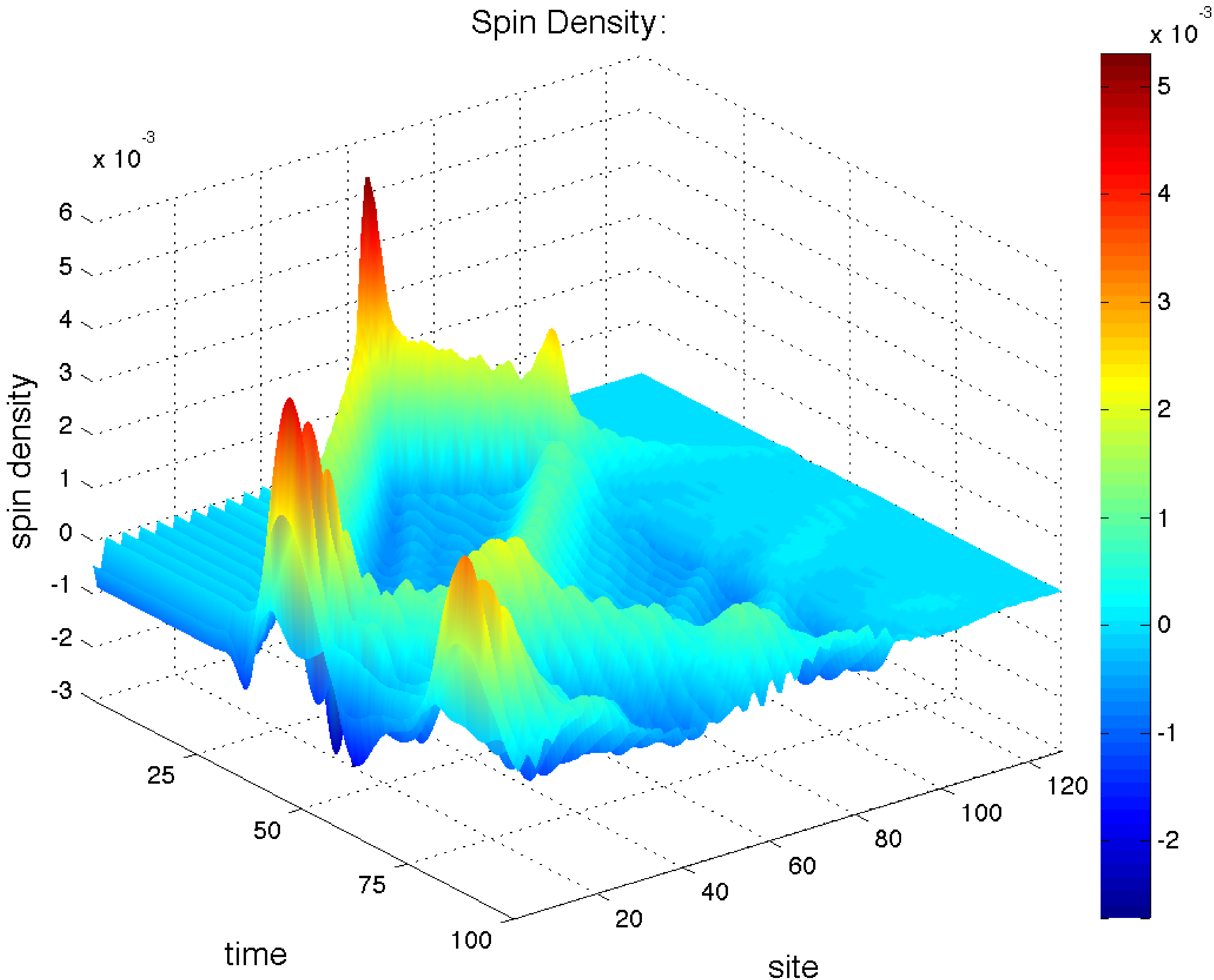
See also Al Hassanieh '15 (Mott)

Conclusions

- **Local quantum quenches** in the XXZ model
  - Bound string states appear prominently,
    both in the ferromagnet and in the antiferromagnet at finite magnetization
    Agree precisely with Bethe ansatz calculations
  - Accessible to experiment

- **Scattering of bound states:**
  - Particle-hole conversion, shift of wall by 2 sites, forward jump of signal

- **Andreev-like reflection**



Spin Density:
x 10^-3
6
5
4
3
2
1
0
-1
-2
-3
spin density
25
50
75
100
time
20
40
60
80
100
120
site
x 10^-3
5
4
3
2
1
0
-1
-2

An unexplained identity

1) Tight binding fermions $H = -\sum_i c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i$

Initial state: domain wall: all sites n<n0 occupied

An ~~unexplained~~ identity

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2) Transverse Field Ising: $\hat{H} = -\sum_n \hat{S}_n^x \hat{S}_{n+1}^x - h \sum_n \hat{S}_n^z$ (at h<h_c=0.5)

(Can be solved by Jordan-Wigner-Flip and Bogoliubov Transformation)

Initial state: prepare symm. broken ground state $|\Downarrow\rangle$ with $\langle S_n^x \rangle < 0$

Then apply a “Jordan-Wigner-Flip” $(c_{n_0}^\dagger + c_{n_0})\,|\Downarrow\rangle = \prod_{n<n_0} (-2\hat{S}_n^z)(2\hat{S}_{n_0}^x)\,|\Downarrow\rangle$

(domain wall in x-direction, + spin flip in z at n_0)

An unexplained identity

Explanation (proof) of the identity: V. Eisler, M. Maislinger, H.G. Evertz, SciPost Phys. 1, 014 (2016)

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(domain wall in x-direction, + spin flip in z at n_0)

3) Find $[S^x(n,t) - S_{\mathrm{GS}}^x]/|2S_{\mathrm{GS}}^x| = N_{\mathrm{TB}}(n, vt)$ (v=h) to 8 digit precision.

Why ?