Hybrid Fokker-Planck Monte Carlo Techniques

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- Introduction
- Hybrid Fokker-Planck Monte Carlo techniques
- Application of hybrid techniques on:
  - NBI slowing down
  - NBCD, v_{tor}
  - ECRH, ECCD
- Discussion

Why combining Fokker-Planck and Monte Carlo?

$$f = f_{\mathrm{M}} + \delta f_{\mathrm{FP}} + \delta f_{\mathrm{MC}}$$

Advantages:

Fokker-Planck (δf_{FP}):

- Highly localized in small phase space volumes (ECRH)
- Nonlinear treatment of collision operator

Monte Carlo (δf_{MC}):

- Global and complex processes, high-dimensional problems
- Switching of co-ordinates

Disadvantages:

Fokker-Planck (δf_{FP}):

- High-dimensional problems (neoclassical diffusion)
- Only asymptotic solutions (collisionless and collisional approach)

Monte Carlo (δf_{MC}):

- Conservation of momentum and energy in collisions

Hybrid Fokker-Planck Monte Carlo techniques I

$$\underbrace{V(f)}_{\text{first order}} = \underbrace{C(f) + Q(f)}_{\text{diffusive}} + \underbrace{S(f)}_{\text{sources}} + \underbrace{L(f)}_{\text{sinks}}$$

- Ansatz: $f = \bar{f} + \delta f_{\mathrm{MC}}$, $\bar{f} = f_{\mathrm{M}} + \delta f_{\mathrm{FP}}$

$$V(\delta f_{\mathrm{MC}}) - C(\delta f_{\mathrm{MC}}) - Q(\delta f_{\mathrm{MC}}) - S(\delta f_{\mathrm{MC}}) - L(\delta f_{\mathrm{MC}}) = \\ -V(\bar{f}) + C(\bar{f}) + Q(\bar{f}) + S(\bar{f}) + L(\bar{f})$$

- Trying to find a Fokker-Planck model equation with $\|\bar{f}\| \gg \|\delta f_{\mathrm{MC}}\|$:

$$\langle C(\bar{f})\rangle_b + \langle Q(\bar{f})\rangle_b + \langle S(\bar{f})\rangle_b = L_{\mathrm{FP}} \cdot \bar{f}$$

- $\langle \ldots \rangle_b$ bounce or flux-surface averaging
- L_{FP} used for particle and power balance (e.g. by $\dot{n}$ and $\dot{T}$ modelling)

Hybrid Fokker-Planck Monte Carlo techniques II

- Introduce *small* quantities

$$\delta C(\bar{f}) = C(\bar{f}) - \langle C(\bar{f})\rangle_b; \quad \delta Q(\bar{f}) = Q(\bar{f}) - \langle Q(\bar{f})\rangle_b$$

$$\delta S(\bar{f}) = S(\bar{f}) - \langle S(\bar{f})\rangle_b$$

- Setting up a δf Monte Carlo technique

$$V(\delta f_{\rm MC}) - C(\delta f_{\rm MC}) - Q(\delta f_{\rm MC}) - S(\delta f_{\rm MC}) - L(\delta f_{\rm MC}) = \\ -V(\bar{f}) + L_{\rm FP}\bar{f} + \underbrace{L(\bar{f}) + \delta C(\bar{f}) + \delta Q(\bar{f}) + \delta S(\bar{f})}_{typically\ neglected}$$

- Marker equation (inhomogeneity $\rightarrow$ characteristic equations)

$$w = \int \left(-V(\bar{f}) + L_{\rm FP}\bar{f}\right) dt = \int F(r,p,v)dt$$

Hybrid NBI slowing down I

- Simulation of slowing-down behavior of suprathermal particles
- W7-X: particle remain near their “birth” flux surface (quasi-isodynamical concept)
- Slowing-down on flux surfaces (Fokker-Planck) delivers a good estimate (most of the physics included)
- Monte Carlo corrects the Fokker-Planck solution regarding to phase space diffusion

Hybrid NBI slowing down II

- Linearized collision term

$$\begin{aligned} C(f,f) &= C(\text{integral part},\ \text{differential part}) \\ &= C(\text{field particles},\ \text{test particles}) \end{aligned}$$

- Fokker-Planck:

$$\langle C(\delta f_{\rm FP}, f_{\rm M})\rangle + \langle C(f_{\rm M}, \delta f_{\rm FP})\rangle + \langle S_{\rm NBI}\rangle = L_{\rm FP}\cdot \bar{f}$$

- $S_{\rm NBI}$ NBI source (e.g. birth profile), $L_{\rm FP}$ loss term ($\dot{n}$, $\dot{T}$ model)
- Monte-Carlo:

$$V(\delta f_{\rm MC}) - C(\bar{f}, \delta f_{\rm MC}) - \cancel{C(\delta f_{\rm MC}, \bar{f})} = -V(\bar{f}) + L_{\rm FP}\bar{f}$$

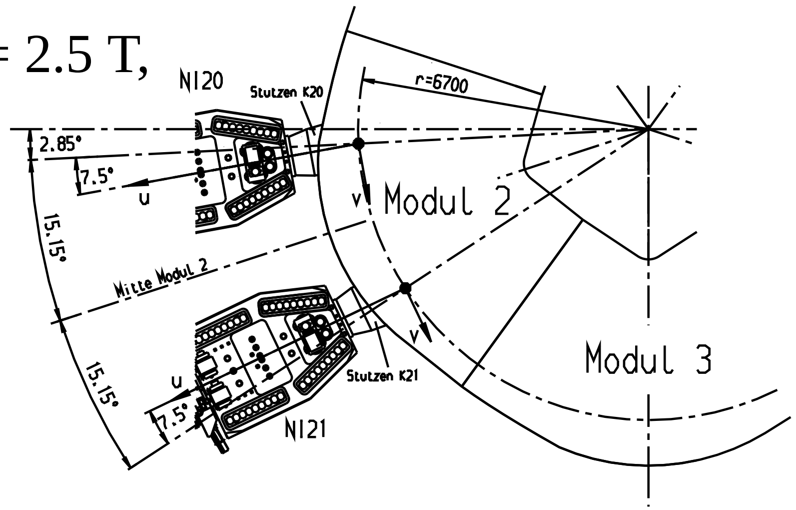
$$\Downarrow$$

Simulation time $t_s \ll$ Slowing down time t_{sd}

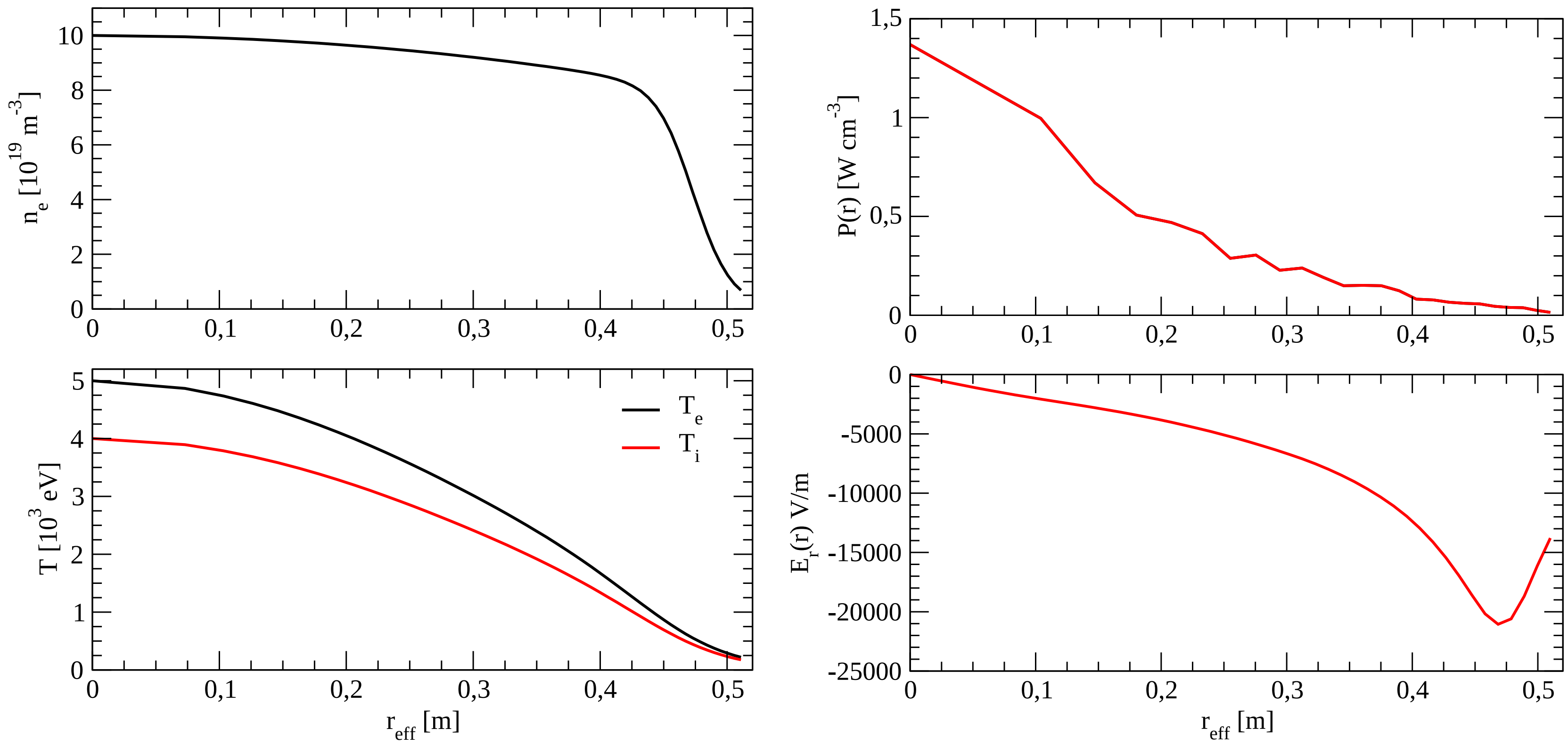
W7-X Example

Discharge Parameters

- $H^0 \rightarrow H^+$ injection with 4 beams of 1.8 MW,
- mean pitch angle of ± 65,
- injection energy of 55 KeV,
- average magnetic field stength B= 2.5 T,
- $Z_{\text{eff}} \approx 2$



W7-X Example



n_e [10^19 m^-3]
10
8
6
4
2
0
0
0,1
0,2
0,3
0,4
0,5
P(r) [W cm^-3]
1,5
1
0,5
0
T [10^3 eV]
5
4
3
2
1
T_e
T_i
r_eff [m]
E_r(r) V/m
-5000
-10000
-15000
-20000
-25000

W7-X Example - Simulation Results

Total bootstrap currents:

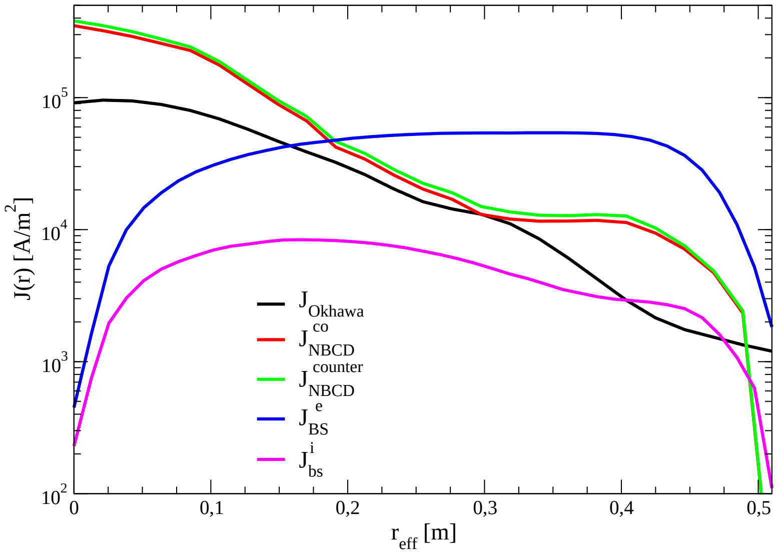
Electrons: 33.3 kA

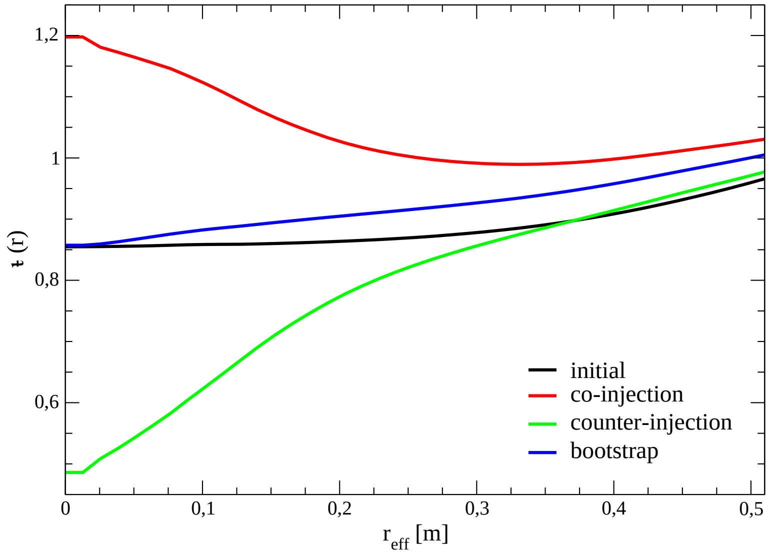
Ions: 3.4 kA

Total NBI driven currents:

counter-injection: 26.3 kA

co-injection: 24.2 kA

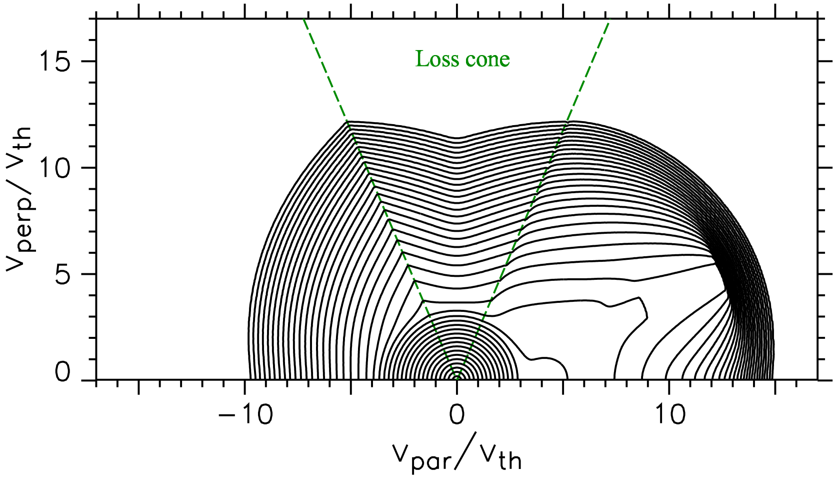




obtained with the *Predictive Transport Code* of Yuriy A. Turkin

Hybrid for NBCD, $v_{\rm tor}$ I

- Strong toroidal velocity $v_{\rm tor}$ (NCSX)



- Fokker-Planck:

$$\bar{f} = f_{pp}^{+} + f_{pp}^{-} + f_{tp}^{\nu}$$

$$\langle C(f_{pp}^{\pm}, f_{pp}^{\pm})\rangle_b + \langle C(f_{pp}^{\pm}, f_{tp}^{\nu})\rangle_b + \langle C(f_{tp}^{\nu}, f_{pp}^{\pm},)\rangle_b + \langle S_{\rm NBI}\rangle_b = L_{\rm FP} \cdot \bar{f}$$

- $S_{\rm NBI}$ NBI source (e.g. birth profile), $L_{\rm FP}$ loss term ($\dot{n}$, $\dot{T}$ model)

Hybrid for NBCD, $v_{\rm tor}$ II

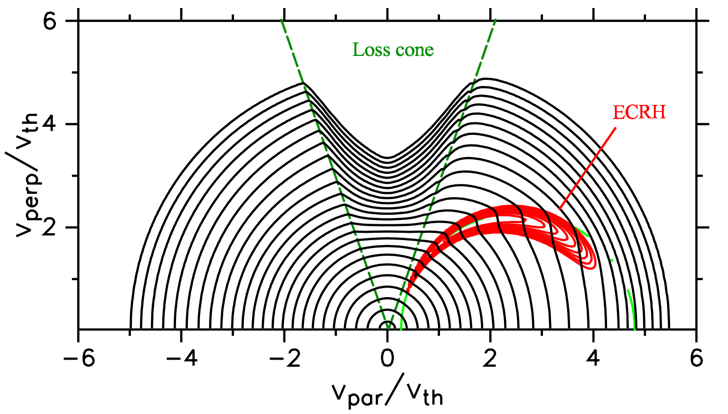
- Legendre expansion of $C(f,f) \approx C(\sum_n P_n f_n, f)$:
  - 0th order: particle balance, power balance
  - 1st order: momentum balance,
  - n-th order: if strong $v_{\rm tor}$ ...
- Monte-Carlo:

$$V(\delta f_{\rm MC}) - C(\bar{f}, \delta f_{\rm MC}) - \underbrace{C(\delta f_{\rm MC}, \bar{f})}_{?} = -V(\bar{f}) + L_{\rm FP}\bar{f}$$

- if $\delta f_{\rm FP}$ is assymetric then $C(\delta f_{\rm MC}, \delta f_{\rm FP})$ is a source respectivly a sink of momentum

Hybrid for ECRH

- Heating is highly localized in phase space



- Fokker-Planck:

$$\langle C(\delta f_{\mathrm{FP}}, f_{\mathrm{M}})\rangle + \langle C(f_{\mathrm{M}}, \delta f_{\mathrm{FP}})\rangle + \langle Q^{\mathrm{ql}}(\bar{f})\rangle_b = L_{\mathrm{FP}} \cdot \bar{f}$$

- ECRH source, e.g. $Q^{\mathrm{ql}} = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp Q_{\perp\perp}\frac{\partial}{\partial v_\perp}$, L_{FP} loss term ($\dot{T}$ model)

- Monte-Carlo:

$$V(\delta f_{\mathrm{MC}}) - C(\bar{f}, \delta f_{\mathrm{MC}}) - \underbrace{C(\delta f_{\mathrm{MC}}, \bar{f})}_{?} = -V(\bar{f}) + L_{\mathrm{FP}}\bar{f}$$

Discussion

- Fokker-Plank:
  - Delivers good estimates of f in velocity space
  - Nonlinear treatment of collision term possible
- Monte Carlo:
  - Corrects the neglected Vlasow part of the Fokker-Planck approach

Why not using the advantages of both methods???