Drift wave instability excited within the resonant zone by rotating perturbation fields*

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- TEXTOR
- Dynamic Ergodic Divertor
- Dielectric tensor
  - Hamiltonian dynamics
  - Canonical perturbation theory
- Drift waves driven by temperature gradients

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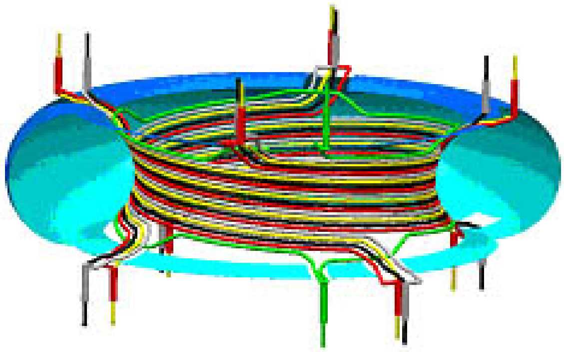

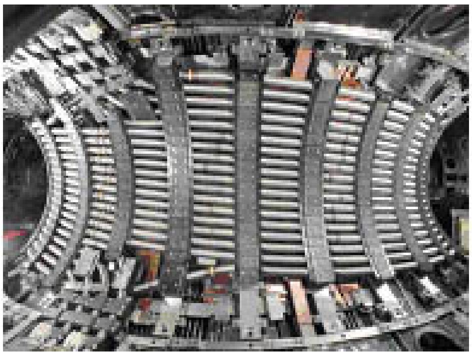

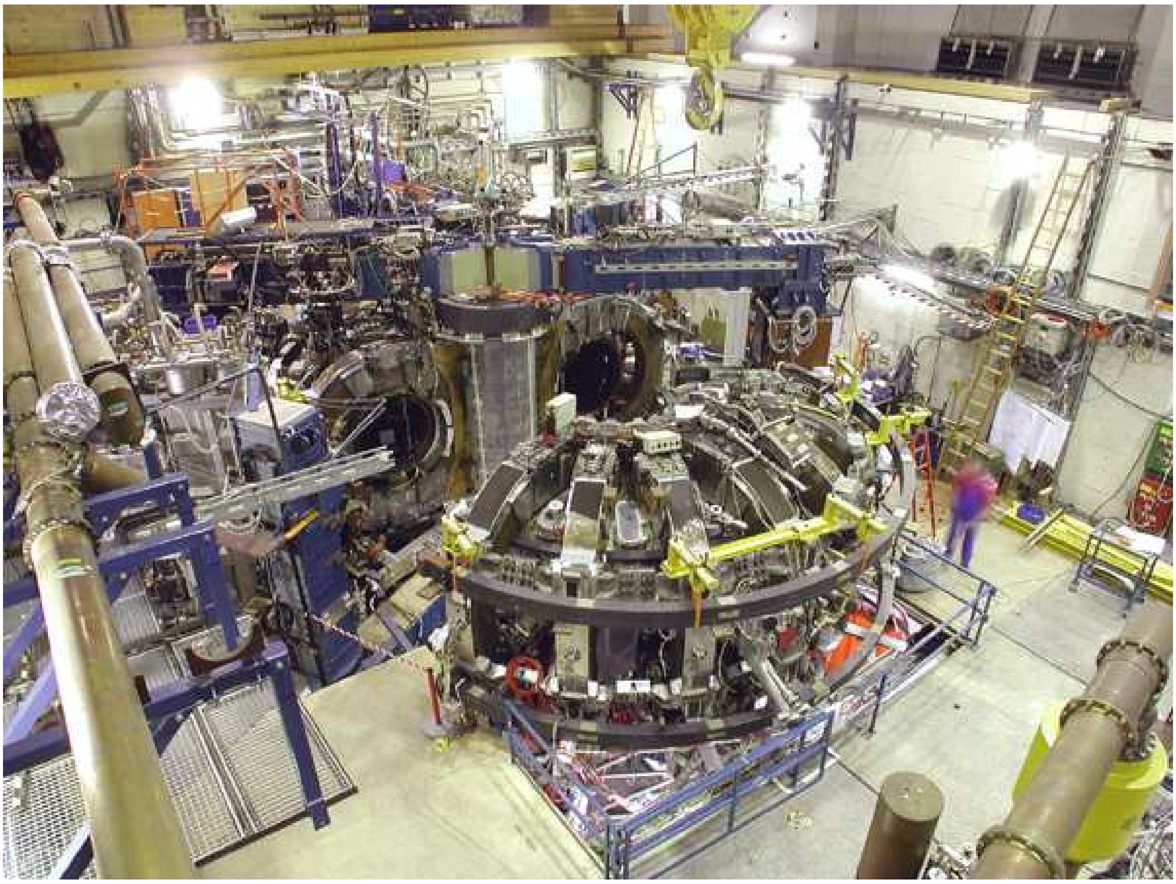

Figure 1: TEXTOR.

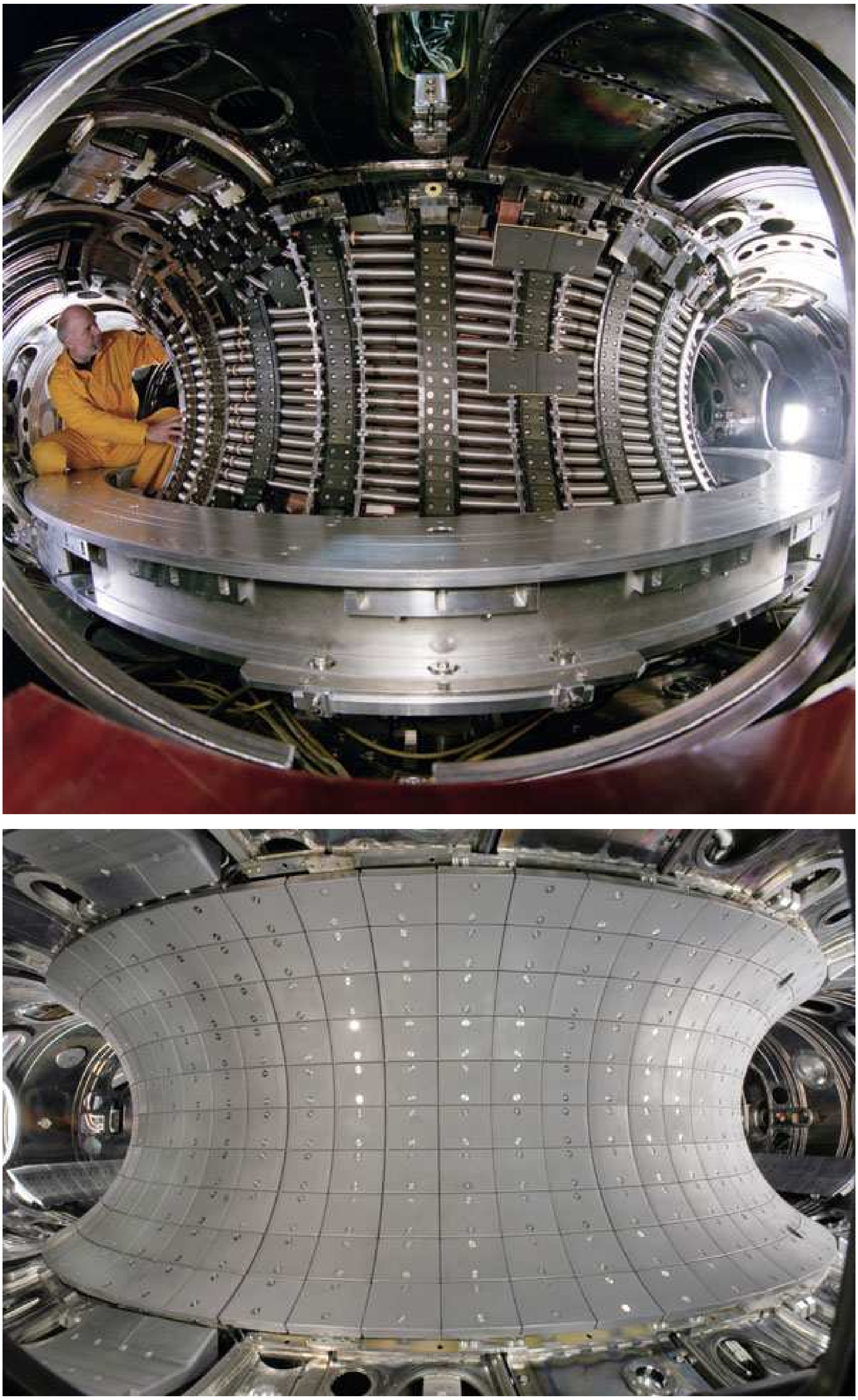

Figure 2: Dynamic Ergodic Divertor Coils.

1 Hamiltonian Dynamics

Hamiltonian of a charged particle (radiation gauge):

$$
\begin{aligned}
\mathbf{B}(\mathbf{q},t) &= \mathrm{rot}\,\left(\mathbf{A}_0(r)+\tilde{\mathbf{A}}(\mathbf{q},t)\right), \\
\mathbf{E}(\mathbf{q},t) &= -\mathrm{grad}\,\Phi_0(r)-\frac{1}{c}\frac{\partial}{\partial t}\tilde{\mathbf{A}}(\mathbf{q},t), \\
H(\mathbf{p},\mathbf{q},t) &= H_0(\mathbf{p},r)+\tilde{H}(\mathbf{p},\mathbf{q},t)+\mathcal{O}\left(\tilde{\mathbf{A}}^2\right) \\
&= \frac{1}{2m}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}_0(r)\right]^2+e\Phi_0(r) \\
&\quad+\frac{1}{2m}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}_0(r)\right]\cdot\frac{e}{c}\tilde{\mathbf{A}}(\mathbf{q},t)+\mathcal{O}\left(\tilde{\mathbf{A}}^2\right),
\end{aligned}
$$

$\mathbf{q} = (r,\theta,z)$… generalized coordinates,

$\mathbf{p} = m\mathbf{v}+\frac{e}{c}\left[\mathbf{A}_0(r)+\tilde{\mathbf{A}}(\mathbf{q},t)\right]$ … canonical momenta.

Evolution of the distribution function $f(\mathbf{p},\mathbf{q},t)$:

$$
\begin{aligned}
\frac{\partial f}{\partial t}+\{f,H\} &= 0, \\
\{f,H\} &:= \frac{\partial f}{\partial \mathbf{q}}\cdot\frac{\partial H}{\partial \mathbf{p}}-\frac{\partial H}{\partial \mathbf{q}}\cdot\frac{\partial f}{\partial \mathbf{p}}.
\end{aligned}
$$

Action-angle variables $(\mathbf{J},\boldsymbol{\Theta})$ and Fourier expanding:

$$
\frac{\partial H_0}{\partial \mathbf{J}} = \boldsymbol{\Omega}_0, \qquad \tilde{H} = \sum_{\mathbf{m}} H_{\mathbf{m}}\,\mathrm{e}^{i\mathbf{m}\cdot\boldsymbol{\Theta}-i\omega t}, \qquad \tilde{f} = \sum_{\mathbf{m}} f_{\mathbf{m}}\,\mathrm{e}^{i\mathbf{m}\cdot\boldsymbol{\Theta}-i\omega t},
$$

$$
f_{\mathbf{m}} = \frac{H_{\mathbf{m}}}{\mathbf{m}\cdot\boldsymbol{\Omega}_0-\omega}\,\mathbf{m}\cdot\frac{\partial f_0}{\partial \mathbf{J}}.
$$

2 Current density

$$
\begin{aligned}
\mathbf{j}(\mathbf{r},t) &= \int \mathrm{d}^3 p_k\, e\mathbf{v} f \;=\; \int \mathrm{d}^3 p \int \mathrm{d}^3 r'\, \delta(\mathbf{r}-\mathbf{r}')\frac{e}{m}\left(\mathbf{p}-\frac{e}{c}\left(\mathbf{A}_0(\mathbf{r}')+\tilde{\mathbf{A}}\right)\right)\left(f_0+\tilde{f}\right) \\
&= \mathbf{j}_0(r) + \frac{i\omega_p^2}{4\pi\omega}\tilde{\mathbf{E}} + \frac{e}{m}\int \mathrm{d}^3 J \int \mathrm{d}^3\Theta\, \delta\left(\mathbf{r}-\mathbf{r}'\left(\mathbf{J},\boldsymbol{\Theta}\right)\right) \\
&\quad \times \left(\mathbf{p}\left(\mathbf{J},\boldsymbol{\Theta}\right) - \frac{e}{c}\mathbf{A}_0\left(\mathbf{J},\boldsymbol{\Theta}\right)\right)\tilde{f}\left(\mathbf{J},\boldsymbol{\Theta},t\right), \\
n_0(\mathbf{r}) &= \int \mathrm{d}^3 J \int \mathrm{d}^3\Theta\, \delta\left(\mathbf{r}-\mathbf{r}'\left(\mathbf{J},\boldsymbol{\Theta}\right)\right) f_0(\mathbf{J}).
\end{aligned}
$$

3 Background

cylindrical geometry with rotational transform

$$
H_0 = \frac{p_r^2}{2m} + \frac{1}{2mr^2}\left(p_\theta - \frac{e}{c}A_{0\theta}(r)\right)^2 + \frac{1}{2m}\left(p_z - \frac{e}{c}A_{0z}(r)\right)^2 + e\Phi_0(r),
$$

field lines: R_0 . . . big radius, q . . . safety factor.

$$
\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{B_0^\theta}{B_0^z} = -\frac{\mathrm{d}A_{0z}}{\mathrm{d}A_{0\theta}} = \frac{1}{qR_0},
$$

gyromotion:

$$
H_0 = \frac{p_r^2}{2m} + U(r,p_\theta,p_z), \qquad \frac{\partial U(r_0)}{\partial r_0} = 0, \qquad \Longrightarrow \qquad r_0 = r_0\left(p_\theta,p_z\right).
$$

4 Action-angle variables

H_0 does not depend on time and the generalized variables θ and $z \Longrightarrow H_0$, p_θ, and p_z are *constants of motion*.

The unperturbed Hamiltonian is expanded around r_0 and solved in zero order by introducing *action-angle variables*, $(r, \theta, z, p_r, p_\theta, p_z) \rightarrow (J_{\perp 0}, P_\theta, P_z, \Phi_0, \theta_0, z_0)$,

$$\begin{aligned}\bar{H}_0 &= \frac{p_r^2}{2m} + U(r_0, p_\theta, p_z) + \frac{1}{2}\frac{\partial^2 U(r_0, p_\theta, p_z)}{\partial r_0^2}(r - r_0)^2 \\ &= \frac{p_r^2}{2m} + U_0(p_\theta, p_z) + \frac{1}{2}U_0''(p_\theta, p_z)(r - r_0)^2 .\end{aligned}$$

$$J_{\perp 0} = \frac{1}{\pi}\int_{r_{\min}}^{r_{\max}} \mathrm{d}r\, p_r\left(\bar{H}_0, r\right) = \frac{m\left(\bar{H}_0 - U_0\right)}{\sqrt{mU_0''}},$$

$$\bar{H}_0 = U_0 + \sqrt{\frac{U_0''}{m}}J_{\perp 0} = U_0 + \Omega_0 J_{\perp 0}, \qquad \Omega_0 = \sqrt{\frac{U_0''}{m}},$$

canonical momenta:

$$p_r = \sqrt{2m\Omega_0 J_{\perp 0} - m^2\Omega_0^2 (r - r_0)^2}, \qquad p_\theta = P_\theta, \qquad p_z = P_z.$$

generating function F:

$$F(\mathbf{P}, \mathbf{q}) = P_\theta \theta + P_z z + \int_{r_{\text{extr}}}^{r} \mathrm{d}r' \sqrt{2m\Omega_0 J_{\perp 0} - m^2\Omega_0^2 (r' - r_0)^2},$$

angle variables:

$$\begin{aligned}\phi_0 &= \frac{\partial F}{\partial J_{\perp 0}} \quad \Longrightarrow \quad r = r_0 - \sqrt{\frac{2J_{\perp 0}}{m\Omega_0}}\cos\phi_0. \\ \theta_0 &= \frac{\partial F}{\partial P_\theta} = \theta - \sqrt{2m\Omega_0 J_{\perp 0}}\frac{\partial r_0}{\partial P_\theta}\sin\phi_0 - \frac{J_{\perp 0}}{2\Omega_0}\frac{\partial \Omega_0}{\partial P_\theta}\sin 2\phi_0. \\ z_0 &= \frac{\partial F}{\partial P_z} = z - \sqrt{2m\Omega_0 J_{\perp 0}}\frac{\partial r_0}{\partial P_z}\sin\phi_0 - \frac{J_{\perp 0}}{2\Omega_0}\frac{\partial \Omega_0}{\partial P_z}\sin 2\phi_0.\end{aligned}$$

H_0 and p_r in terms of the new variables:

$$\begin{aligned}H_0 &= U_0 + \Omega_0 J_{\perp 0} - \frac{1}{6}U_0'''\left(\frac{2J_{\perp 0}}{m\Omega_0}\right)^{3/2}\cos^3\phi_0 + \frac{1}{24}U_0^{IV}\left(\frac{2J_{\perp 0}}{m\Omega_0}\right)^2\cos^4\phi_0. \\ p_r &= \sqrt{2m\Omega_0 J_{\perp 0}}\sin\phi_0.\end{aligned}$$

5 Canonical perturbation theory

The action-angle variables are transformed with the help of the generating function F

$$
\begin{aligned}
F &= P_\theta \theta_0 + P_z z_0 + J_{\perp 1}\phi_0 + g\left(J_{\perp 1}, P_\theta, P_z; \phi_0\right), \\
J_{\perp 0} &= \frac{\partial F}{\partial \phi_0} = J_{\perp 1} + \frac{\partial g}{\partial \phi_0}, \\
\phi_1 &= \frac{\partial F}{\partial J_{\perp 1}} = \phi_0 + \frac{\partial g}{\partial J_{\perp 1}}, \\
\theta_1 &= \theta_0 + \frac{\partial g}{\partial P_\theta} = \theta_0 + \mathcal{O}\left(\left(\frac{\rho}{r_0}\right)^3\right), \\
z_1 &= z_0 + \frac{\partial g}{\partial P_z} = z_0 + \mathcal{O}\left(\left(\frac{\rho}{r_0}\right)^3\right),
\end{aligned}
$$

$$
\begin{aligned}
H_0 &= U_0 + \Omega_0 \left(J_{\perp 1} + \frac{\partial g}{\partial \phi_0}\right) - \frac{U_0''}{6}\left(\frac{2J_{\perp 1}}{m\Omega_0}\right)^{3/2}\left(1 + \frac{1}{J_{\perp 1}}\frac{\partial g}{\partial \phi_0}\right)^{3/2}\cos^3\phi_0 \\
&\quad + \frac{U_0^{IV}}{24}\left(\frac{2J_{\perp 1}}{m\Omega_0}\right)^2\left(1 + \frac{1}{J_{\perp 1}}\frac{\partial g}{\partial \phi_0}\right)^2\cos^4\phi_0,
\end{aligned}
$$

such that the new Hamiltonian takes the form

$$
\begin{aligned}
H_0 &= U_0 + \Omega_0 J_{\perp 1} - \frac{U_0''}{6}\left(\frac{2J_{\perp 1}}{m\Omega_0}\right)^{3/2}\left(\frac{1}{J_{\perp 1}}\frac{\partial g}{\partial \phi_0}\right)^{3/2}\cos^3\phi_0 \\
&\quad + \frac{U_0^{IV}}{24}\left(\frac{2J_{\perp 1}}{m\Omega_0}\right)^2\left(1 + \frac{1}{J_{\perp 1}}\frac{\partial g}{\partial \phi_0}\right)^2\cos^4\phi_0.
\end{aligned}
$$

This procedure is repeated (second order in ρ/r_0) and the Hamiltonian as well as the "old" variables are expressed through the *new variables*:

$$H_0 = U_0 + \Omega_0 J_{\perp 2} - \frac{5}{48}\frac{\left(U_0'''\right)^2 J_{\perp 2}^2}{m^3\Omega_0^4} + \frac{U_0^{IV} J_{\perp 2}^2}{16 m^2 \Omega_0^2},$$

$$\begin{aligned}\phi_0\frac{1}{2} &= \phi_2 - \frac{U_0'''}{8m\Omega_0^2}\left(\frac{2J_{\perp 2}}{m\Omega_0}\right)^{1/2}\left(3\sin\phi_2 + \frac{1}{3}\sin 3\phi_2\right) \\ &+ \frac{\left(U_0'''\right)^2 J_{\perp 2}^2}{96 m^3\Omega_0^5}\left(-\frac{9}{2}\sin 2\phi_2 + 3\sin 4\phi_2 + \frac{1}{6}\sin 6\phi_2\right) \\ &+ \frac{U_0^{IV} J_{\perp 2}}{24 m\Omega_0^3}\left(2\sin 2\phi_2 + \frac{1}{4}\sin 4\phi_2\right).\end{aligned}$$

$$\begin{aligned}J_{\perp 0} &= J_{\perp 2} + \frac{U_0'''}{24\Omega_0}\left(\frac{2J_{\perp 2}}{m\Omega_0}\right)^{3/2}\left(3\cos\phi_2 + \cos 3\phi_2\right) \\ &+ \frac{\left(U_0'''\right)^2 J_{\perp 2}^2}{48 m^3\Omega_0^5}\left(5 + 8\cos 2\phi_2 - 2\cos 4\phi_2\right) \\ &- \frac{U_0^{IV} J_{\perp 2}^2}{48 m^2\Omega_0^3}\left(4\cos 2\phi_2 + \cos 4\phi_2\right).\end{aligned}$$

$$\begin{aligned}r - r_0 &= -\rho\cos\phi_2 - \frac{U_0'''\rho^2}{4m\Omega_0^2}\left(1 - \frac{1}{3}\cos 2\phi_2\right) \\ &+ \frac{U_0^{IV}\rho^3}{48 m\Omega_0^3}\left(\frac{3}{2}\cos\phi_2 - \frac{1}{4}\cos 3\phi_2\right) \\ &- \frac{\left(U_0'''\right)^2\rho^3}{48 m^2\Omega_0^4}\left(\frac{11}{6}\cos\phi_2 + \frac{1}{4}\cos 3\phi_2\right), \\ \theta &= \theta_2 + m\Omega_0\frac{\partial r_0}{\partial p_\theta}\rho\sin\Phi_2 - \frac{U_0'''\rho^2}{6\Omega_0}\frac{\partial r_0}{\partial p_\theta}\sin 2\Phi_2 + \frac{1}{4}m\frac{\partial\Omega_0}{\partial p_\theta}\rho^2\sin 2\Phi_2, \\ z &= z_2 + m\Omega_0\frac{\partial r_0}{\partial p_z}\rho\sin\Phi_2 - \frac{U_0'''\rho^2}{6\Omega_0}\frac{\partial r_0}{\partial p_z}\sin 2\Phi_2 + \frac{1}{4}m\frac{\partial\Omega_0}{\partial p_z}\rho^2\sin 2\Phi_2.\end{aligned}$$

perturbed Hamiltonian and distribution function:

$$\tilde{H} = \frac{ie}{m\omega}\left(p_r \tilde{E}^r + \left(p_\theta - \frac{e}{c}A_{0\theta}\right)\tilde{E}^\theta + \left(p_z - \frac{e}{c}A_{0z}\right)\tilde{E}^z\right)$$

$$\tilde{f} = \underbrace{\tilde{H}\frac{\partial f_0}{\partial \bar{H}_0}}_{\tilde{f}^{(1)}} + \sum_{\mathbf{m}} \underbrace{\mathrm{e}^{i\mathbf{m}\cdot\mathbf{\Theta}-i\omega t}\frac{H_{\mathbf{m}}}{\mathbf{m}\cdot\mathbf{\Omega}_0-\omega}\left(\omega\frac{\partial f_0}{\partial \bar{H}_0} + \mathbf{m}\cdot\frac{\partial r_0}{\partial \mathbf{J}}\frac{\partial f_0}{\partial r_0} + \mathbf{m}\cdot\frac{\partial u_\parallel}{\partial \mathbf{J}}\frac{\partial f_0}{\partial u_\parallel}\right)}_{\tilde{f}^{(2)}_{\bar{m}}}$$

perturbed current deinsity:

$$\begin{aligned}\tilde{j}_i = & \frac{i\omega_p^2}{4\pi\omega}\tilde{E}_i \\ & + \frac{e2\pi}{r}\sum_{\bar{m}=-2}^{2}\int_{-\infty}^{\infty}\mathrm{d}u_\parallel\int_0^\infty \mathrm{d}J_\perp \Bigg\{ \\ & \times\left[(v_i)_{-\bar{m}} - i\left(\mathbf{k}\cdot\Delta\mathbf{r}v_i\right)_{-\bar{m}} - \frac{1}{2}\left((\mathbf{k}\cdot\Delta\mathbf{r})^2 v_i\right)_{-\bar{m}}\right] \\ & - \frac{\partial}{\partial r}\left[(\Delta r v_i)_{-\bar{m}} - i\left(\Delta r\left(\mathbf{k}\cdot\Delta\mathbf{r}\right)v_i\right)_{-\bar{m}}\right] \\ & + \frac{\partial^2}{\partial r^2}\left[\frac{1}{2}\left((\Delta r)^2 v_i\right)_{-\bar{m}}\right]\Bigg\} J\left(\tilde{f}^{(1)} + \tilde{f}^{(2)}_{\bar{m}}\right).\end{aligned}$$

$$\begin{aligned}\tilde{j}_i = & \; s^{11}_{ij}\tilde{E}^j + s^{12}_{ij}\tilde{E}'^j + s^{13}_{ij}\tilde{E}''^j \\ & - \frac{1}{r}\frac{\partial}{\partial r}r\left[s^{21}_{ij}\tilde{E}^j + s^{22}_{ij}\tilde{E}'^j + s^{23}_{ij}\tilde{E}''^j\right] \\ & + \frac{1}{r}\frac{\partial^2}{\partial r^2}r\left[s^{31}_{ij}\tilde{E}^j + s^{32}_{ij}\tilde{E}'^j + s^{33}_{ij}\tilde{E}''^j\right] \\ = & \; \sigma_{ij}\tilde{E}^j + \bar{\sigma}_{ij}\tilde{E}'^j + \bar{\bar{\sigma}}_{ij}\tilde{E}''^j.\end{aligned}$$

power balance:

$$\frac{1}{2}\Re\left\{\frac{1}{r}\frac{\partial}{\partial r}r\frac{c}{4\pi}\mathbf{e}_r\cdot\left[\mathbf{E}^*\times\mathbf{B}\right]\right\} = -\frac{1}{2}\Re\left\{\mathbf{j}^*\cdot\mathbf{E}\right\},$$

$$P_r = \frac{1}{2}\Re\left\{\frac{c}{4\pi}\mathbf{E}^*\times\mathbf{B}\right\},$$

$$F_{\mathrm{mat}\,r} = -\frac{1}{2}\Re\left\{E^{i\star}\left(s^{21}_{ij}E^j + s^{22}_{ij}E'^j\right) + E'^{i\star}s^{31}_{ij}E^j - E^{i\star}\frac{1}{r}\frac{\partial}{\partial r}\left(r s^{31}_{ij}E^j\right)\right\},$$

$$p_{\mathrm{loc}}(r) = \frac{1}{2}\Re\left\{s^{11}_{ij}E^{i\star}E^j + s^{12}E^{i\star}E'^j + s^{31}_{ij}E^{i\star}E''^j + s^{21}_{ij}E'^{i\star}E^j + s^{22}_{ij}E'^{i\star}E'^j + s^{31}_{ij}E''^{i\star}E^j,\right\}.$$

$$\frac{1}{r}\frac{\partial}{\partial r}r\left[P_r + F_{\mathrm{mat}\,r}\right] = p_{\mathrm{loc}}(r),$$

$$\left(2\pi R_0\right)\left(2\pi r\right)\underbrace{\left[P_r + F_{\mathrm{mat}\,r}\right]}_{F_{\mathrm{tot}\,r}} = \left(2\pi R_0\right)\left(2\pi r\right)\int_0^r \mathrm{d}r' r' p_{\mathrm{loc}}(r').$$

```
> print(gsig);
```

array(1 .. 3, 1 .. 3, 1 .. 3, [

$$(1,1,1) = -\frac{\mathrm{om\_D}(r\_0)^2}{\mathrm{om\_c}(r\_0)^2 - \mathrm{om\_D}(r\_0)^2}$$

$$(1,1,2) = -2\,\frac{\mathrm{k\_s}(r\_0)\,\mathrm{om\_c}(r\_0)\,\mathrm{om\_D}(r\_0)\,\mathrm{V\_p}(r\_0)^2\,\mathrm{h\_t}(r\_0)^2}{(-\mathrm{om\_c}(r\_0)+\mathrm{om\_D}(r\_0))^2\,(\mathrm{om\_c}(r\_0)+\mathrm{om\_D}(r\_0))^2\,r\_0}$$

$$+\frac{2\,\mathrm{k\_s}(r\_0)\,\mathrm{om\_c}(r\_0)\,\mathrm{om\_D}(r\_0)\,qe\left(\frac{\partial}{\partial r\_0}\mathrm{Phi\_0}(r\_0)\right)}{(-\mathrm{om\_c}(r\_0)+\mathrm{om\_D}(r\_0))^2\,(\mathrm{om\_c}(r\_0)+\mathrm{om\_D}(r\_0))^2\,m}$$

$$+\frac{2\,\mathrm{k\_s}(r\_0)\,\mathrm{om\_c}(r\_0)\,\mathrm{om\_D}(r\_0)\left(\frac{\partial}{\partial r\_0}\mathrm{p}(r\_0)\right)}{(-\mathrm{om\_c}(r\_0)+\mathrm{om\_D}(r\_0))^2\,(\mathrm{om\_c}(r\_0)+\mathrm{om\_D}(r\_0))^2\,\mathrm{n}(r\_0)\,m}$$

$$(1,1,3) = \Bigg(4\,\frac{\mathrm{om\_D}(r\_0)\,\mathrm{k\_s}(r\_0)}{(-4\,\mathrm{om\_c}(r\_0)^2+\mathrm{om\_D}(r\_0)^2)\,\mathrm{om\_c}(r\_0)}$$

$$+\frac{2\,(-\mathrm{om\_D}(r\_0)\,\mathrm{h\_t}(r\_0)^2+\mathrm{om\_D}(r\_0))\,\mathrm{om\_D}(r\_0)}{\mathrm{om\_c}(r\_0)^2\,(-4\,\mathrm{om\_c}(r\_0)^2+\mathrm{om\_D}(r\_0)^2)\,r\_0}\Bigg)\mathrm{v\_T}(r\_0)\left(\frac{\partial}{\partial r\_0}\mathrm{v\_T}(r\_0)\right)+\Bigg($$

$$\frac{\mathrm{om\_D}(r\_0)^2\,\mathrm{v\_T}(r\_0)^2\left(\frac{\partial}{\partial r\_0}\mathrm{om\_c}(r\_0)\right)}{\mathrm{n}(r\_0)\,\mathrm{om\_c}(r\_0)^3\,(4\,\mathrm{om\_c}(r\_0)^2-\mathrm{om\_D}(r\_0)^2)}+\Bigg($$

$$-2\,\frac{\mathrm{om\_D}(r\_0)\,\mathrm{k\_s}(r\_0)}{\mathrm{om\_c}(r\_0)\,\mathrm{n}(r\_0)\,(4\,\mathrm{om\_c}(r\_0)^2-\mathrm{om\_D}(r\_0)^2)}$$

$$-\frac{(-\mathrm{om\_c}(r\_0)\,\mathrm{om\_D}(r\_0)\,\mathrm{h\_t}(r\_0)^2+\mathrm{om\_c}(r\_0)\,\mathrm{om\_D}(r\_0))\,\mathrm{om\_D}(r\_0)}{r\_0\,\mathrm{n}(r\_0)\,\mathrm{om\_c}(r\_0)^3\,(4\,\mathrm{om\_c}(r\_0)^2-\mathrm{om\_D}(r\_0)^2)}\Bigg)\mathrm{v\_T}(r\_0)^2\Bigg)$$

$$\left(\frac{\partial}{\partial r\_0}\mathrm{n}(r\_0)\right)+\Bigg(\mathrm{om\_D}(r\_0)\,\mathrm{v\_T}(r\_0)\,(-2\,\mathrm{om\_D}(r\_0)^3\,r\_0+8\,\mathrm{om\_c}(r\_0)^2\,\mathrm{om\_D}(r\_0)\,r\_0)$$

$$\left(\frac{\partial}{\partial r\_0}\mathrm{v\_T}(r\_0)\right)\Big/\left(r\_0\,(4\,\mathrm{om\_c}(r\_0)^2-\mathrm{om\_D}(r\_0)^2)^2\,\mathrm{om\_c}(r\_0)^3\right)$$

$$+\frac{8\,\mathrm{om\_D}(r\_0)\,\mathrm{v\_T}(r\_0)^2\left(\frac{\partial}{\partial r\_0}\mathrm{om\_D}(r\_0)\right)}{\mathrm{om\_c}(r\_0)\,(4\,\mathrm{om\_c}(r\_0)^2-\mathrm{om\_D}(r\_0)^2)^2}+\Bigg($$

$$\frac{\mathrm{om\_D}(r\_0)\,(40\,r\_0\,\mathrm{om\_c}(r\_0)^3-6\,r\_0\,\mathrm{om\_D}(r\_0)^2\,\mathrm{om\_c}(r\_0))\,\mathrm{k\_s}(r\_0)}{r\_0\,(4\,\mathrm{om\_c}(r\_0)^2-\mathrm{om\_D}(r\_0)^2)^2\,\mathrm{om\_c}(r\_0)^3}+\mathrm{om\_D}(r\_0)\,($$

$$28\,\mathrm{om\_c}(r\_0)^2\,\mathrm{om\_D}(r\_0)+4\,\mathrm{om\_D}(r\_0)^3\,\mathrm{h\_t}(r\_0)^2-5\,\mathrm{om\_D}(r\_0)^3$$

Figure 3: Conductivity tensor, asymmetric form, acting on E.

6 Maxwell equations

$$
\begin{aligned}
-\frac{c}{i\omega}E_\parallel' + N_\parallel E_r + N_1 E_s - N_2 E_\parallel - B_s &= 0 \quad [1, E_\parallel'],\\
\frac{c}{i\omega}E_s' - N_s E_r + N_3 E_\parallel + N_4 E_s - B_\parallel &= 0 \quad [2, E_s'],
\end{aligned}
$$

$$
\begin{aligned}
-\frac{c}{i\omega}B_\parallel' + N_\parallel B_r + N_1 B_s - N_2 B_\parallel + \varepsilon_{sr}E_r + \varepsilon_{ss}E_s + \varepsilon_{s\parallel}E_\parallel &\\
+\bar\varepsilon_{sr}E_r' + \bar\varepsilon_{ss}E_s' + \bar\varepsilon_{s\parallel}E_\parallel' + \bar{\bar\varepsilon}_{sr}E_r'' + \bar{\bar\varepsilon}_{ss}E_s'' + \bar{\bar\varepsilon}_{s\parallel}E_\parallel'' &= 0 \quad [3, B_\parallel'],\\
\frac{c}{i\omega}B_s' - N_s B_r + N_3 B_\parallel + N_4 B_s + \varepsilon_{\parallel r}E_r + \varepsilon_{\parallel s}E_s + \varepsilon_{\parallel\parallel}E_\parallel &\\
+\bar\varepsilon_{\parallel r}E_r' + \bar\varepsilon_{\parallel s}E_s' + \bar\varepsilon_{\parallel\parallel}E_\parallel' + \bar{\bar\varepsilon}_{\parallel r}E_r'' + \bar{\bar\varepsilon}_{\parallel s}E_s'' + \bar{\bar\varepsilon}_{\parallel\parallel}E_\parallel'' &= 0 \quad [4, B_s'],
\end{aligned}
$$

$$
\begin{aligned}
-\frac{c}{i\omega}E_\parallel'' + N_\parallel' E_r + N_\parallel E_r' + N_1' E_s + N_1 E_s' - N_2' E_\parallel - N_2 E_\parallel' - B_s' &= 0 \quad [5, E_\parallel''],\\
-N_\parallel B_s + N_s B_\parallel + \varepsilon_{rr}E_r + \varepsilon_{rs}E_s + \varepsilon_{r\parallel}E_\parallel &\\
+\bar\varepsilon_{rr}E_r' + \bar\varepsilon_{rs}E_s' + \bar\varepsilon_{r\parallel}E_\parallel' + \bar{\bar\varepsilon}_{rr}E_r'' + \bar{\bar\varepsilon}_{rs}E_s'' + \bar{\bar\varepsilon}_{r\parallel}E_\parallel'' &= 0 \quad [6, E_r''],\\
-N_\parallel E_s + N_s E_\parallel - B_r &= 0 \quad [7, B_r],\\
\frac{c}{i\omega}E_s'' - N_s' E_r - N_s E_r' + N_3' E_\parallel + N_3 E_\parallel' + N_4' E_s + N_4 E_s' - B_\parallel' &= 0 \quad [8, E_s''].
\end{aligned}
$$

variables

1	2	3	4	5	6
E_s	$E_\parallel$	B_s	$B_\parallel$	E_r	E_r'
7	8	9	10	11	12
E_s'	$E_\parallel'$	B_s'	$B_\parallel'$	E_r''	$E_\parallel''$
13	14				
B_r	E_s''				

starting values

$E_\parallel$	$E_\parallel'$
$B_\parallel$	$B_\parallel'$
E_r	E_r'.

7 Antenna relations

Solve the inhomogeneous system with zero input state vector for the derivatives and identify: $\Delta E_\parallel = E'_\parallel$, $\Delta E'_\parallel = E''_\parallel$, ...

$$
\begin{aligned}
-\frac{c}{i\omega}\Delta E_\parallel &= 0 \quad [1, \Delta E_\parallel],\\
\frac{c}{i\omega}\Delta E_s &= 0 \quad [2, \Delta E_s],
\end{aligned}
$$

$$
\begin{aligned}
-\frac{c}{i\omega}\Delta B_\parallel + \bar\varepsilon_{sr}\Delta E_r + \bar\varepsilon_{ss}\Delta E_s + \bar\varepsilon_{s\parallel}\Delta E_\parallel + \bar{\bar\varepsilon}_{sr}\Delta E'_r + \bar{\bar\varepsilon}_{ss}\Delta E'_s + \bar{\bar\varepsilon}_{s\parallel}\Delta E'_\parallel &= \frac{c}{i\omega}\frac{4\pi}{c}j_{as} \quad [3, \Delta B_\parallel],\\
\frac{c}{i\omega}\Delta B_s + \bar\varepsilon_{\parallel r}\Delta E_r + \bar\varepsilon_{\parallel s}\Delta E_s + \bar\varepsilon_{\parallel\parallel}\Delta E_\parallel + \bar{\bar\varepsilon}_{\parallel r}\Delta E'_r + \bar{\bar\varepsilon}_{\parallel s}\Delta E'_s + \bar{\bar\varepsilon}_{\parallel\parallel}\Delta E'_\parallel &= \frac{c}{i\omega}\frac{4\pi}{c}j_{a\parallel} \quad [4, \Delta B_s],
\end{aligned}
$$

$$
\begin{aligned}
-\Delta E'_\parallel + \frac{i\omega}{c}\left(N_\parallel \Delta E_r + N_1\Delta E_s - N_2\Delta E_\parallel - \Delta B_s\right) &= 0 \quad [5, \Delta E'_\parallel],\\
\bar\varepsilon_{rr}\Delta E_r + \bar\varepsilon_{rs}\Delta E_s + \bar\varepsilon_{r\parallel}\Delta E_\parallel + \bar{\bar\varepsilon}_{rr}\Delta E'_r + \bar{\bar\varepsilon}_{rs}\Delta E'_s + \bar{\bar\varepsilon}_{r\parallel}\Delta E'_\parallel &= 0 \quad [6, \Delta E'_r],\\
\Delta B_r &= 0 \quad [7, \Delta B_r],\\
\frac{c}{i\omega}\Delta E'_s - N_s\Delta E_r + N_3\Delta E_\parallel + \Delta N_4 E_s - \Delta B_\parallel &= 0 \quad [8, \Delta E'_s].
\end{aligned}
$$

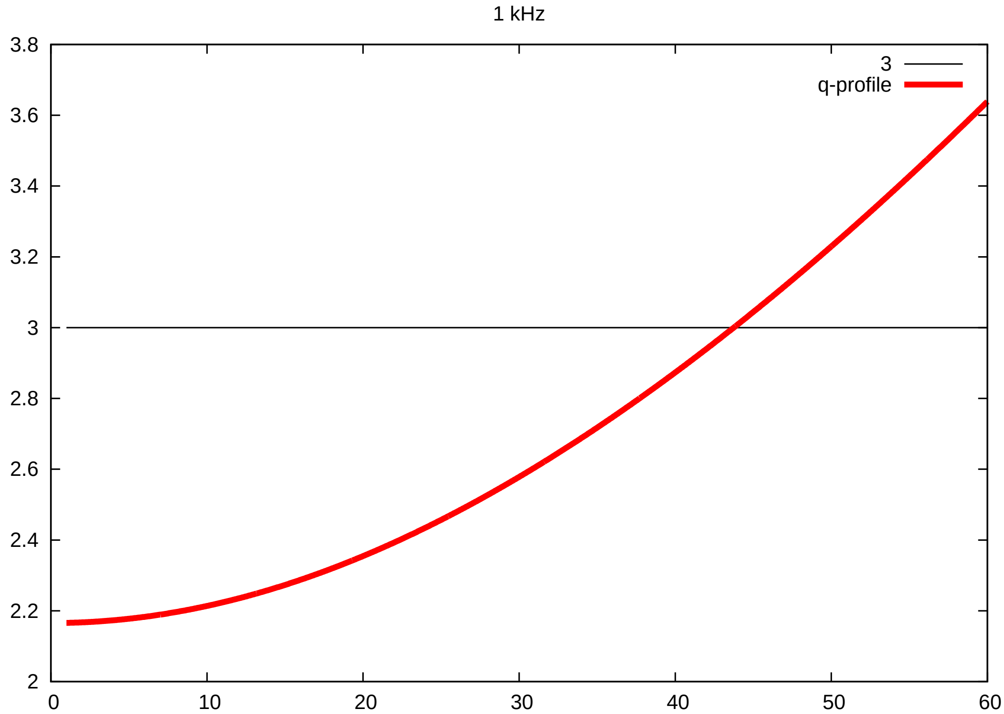

Figure 4: q-factor profile.

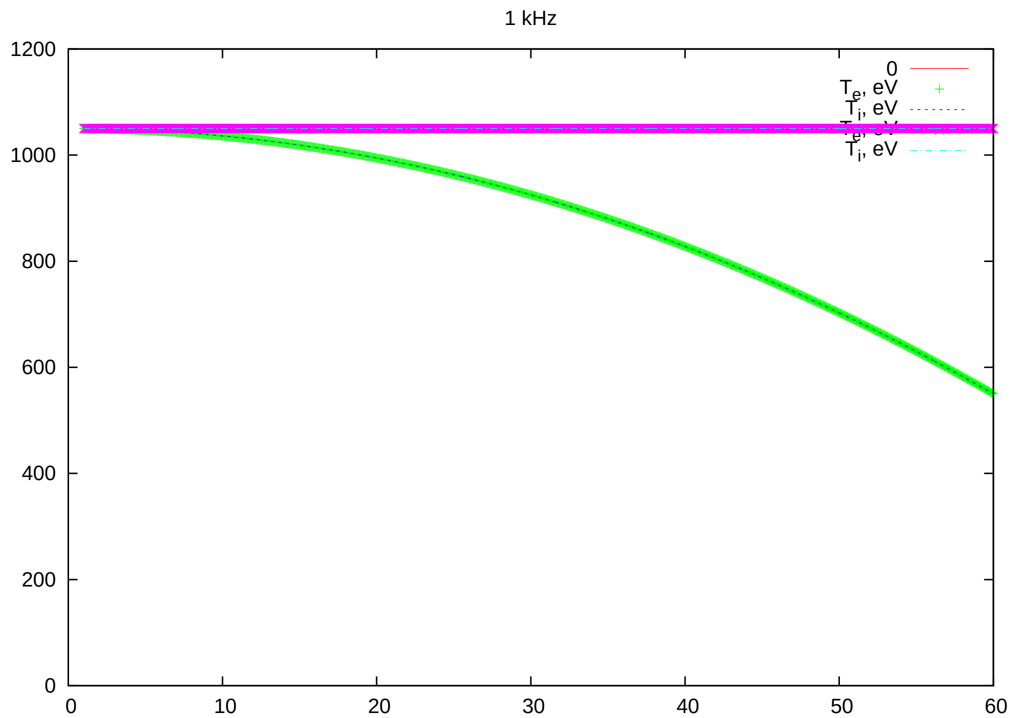

Figure 5: Temperature profiles.

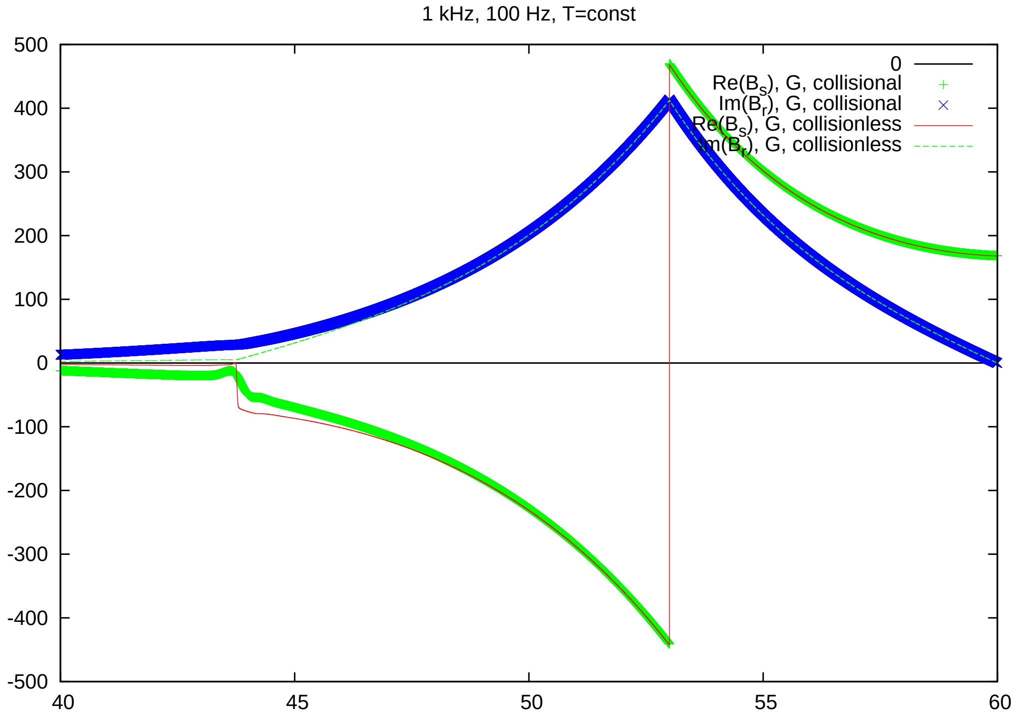

Figure 6: Radial and poloidal magnetic field, $T = const$.

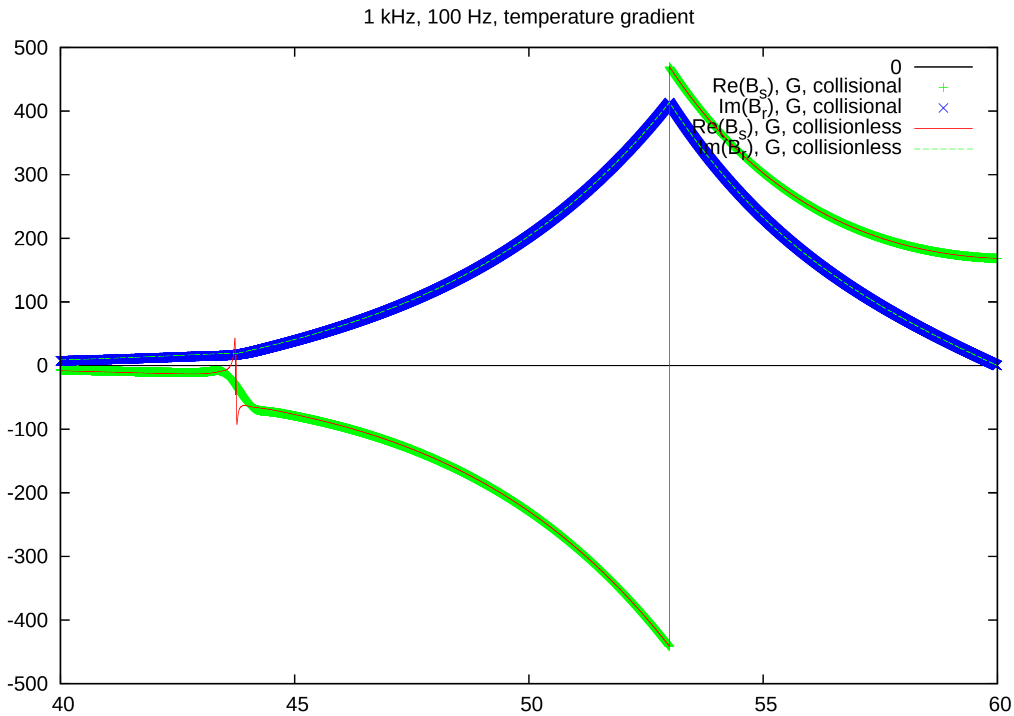

Figure 7: Radial and poloidal magnetic field, temperature gradient.

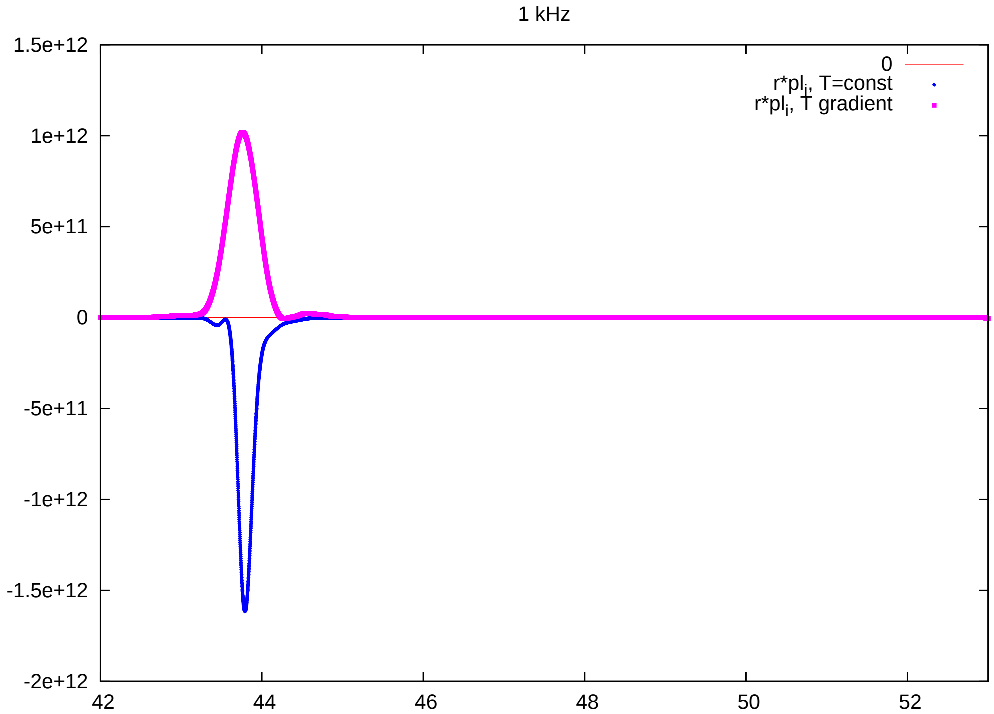

Figure 8: Power deposition due to electrons.

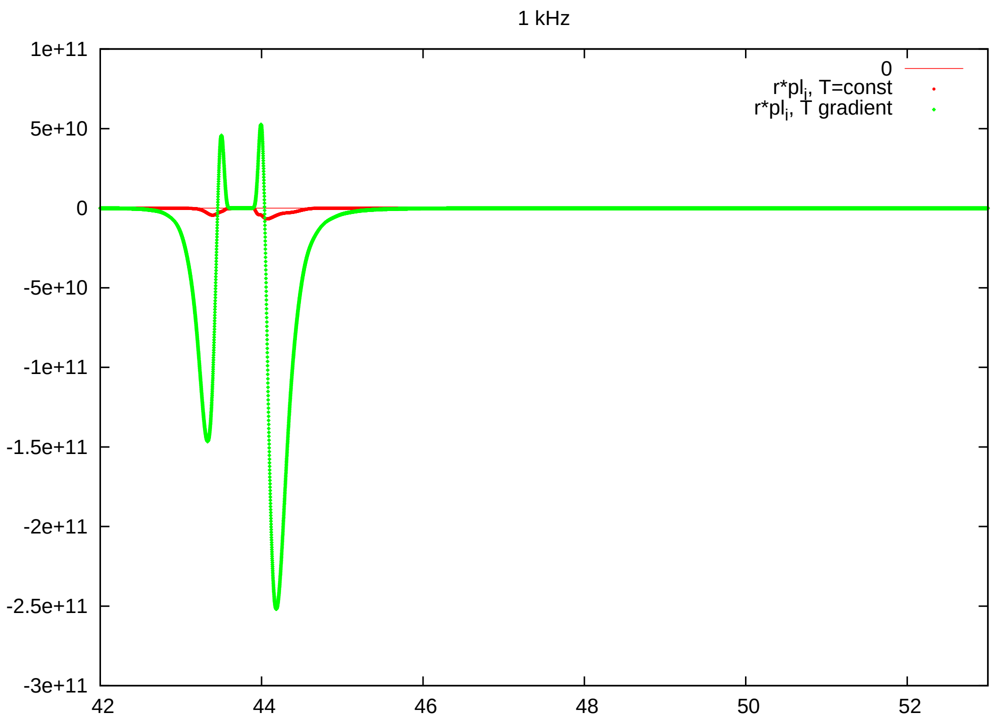

Figure 9: Power deposition due to ions.

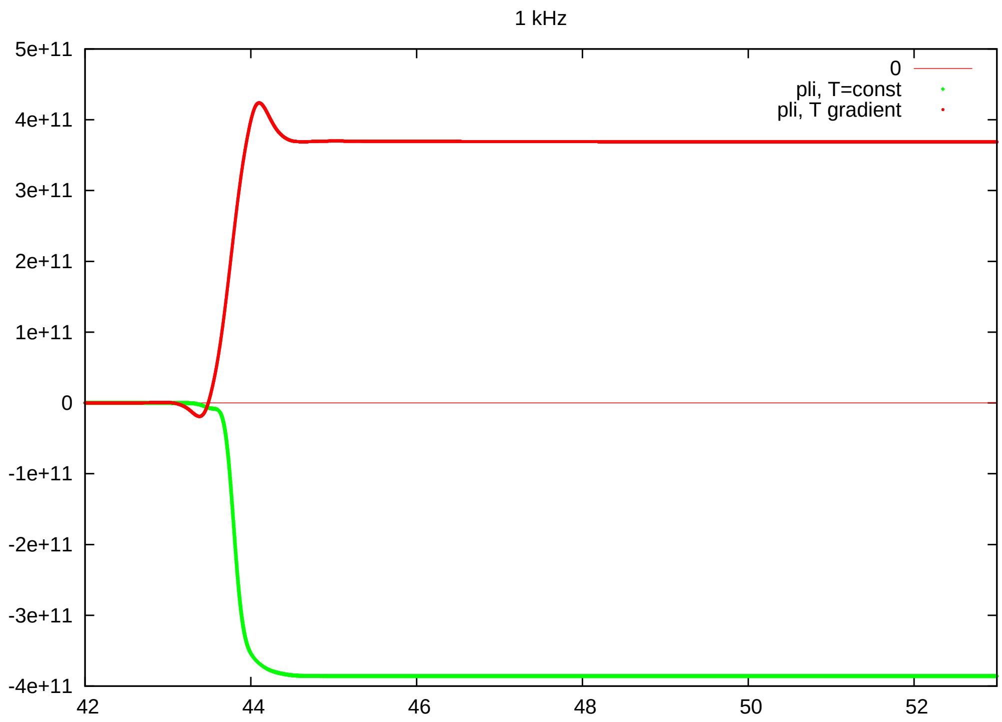

Figure 10: Total energy flux.

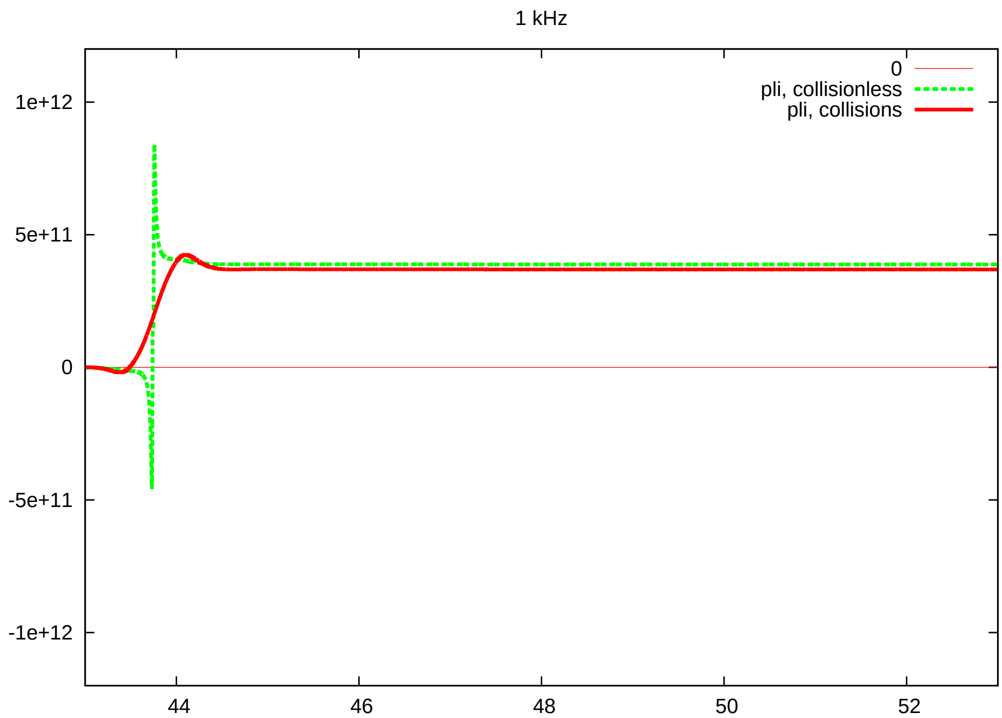

Figure 11: Comparison with collisionless case.

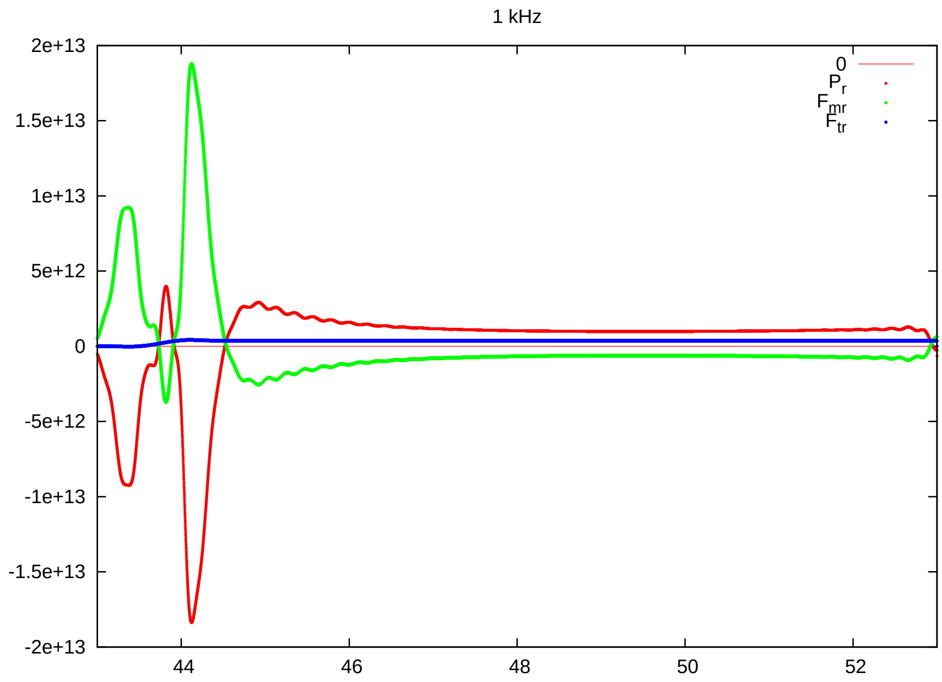

Figure 12: Poynting and material fluxes.

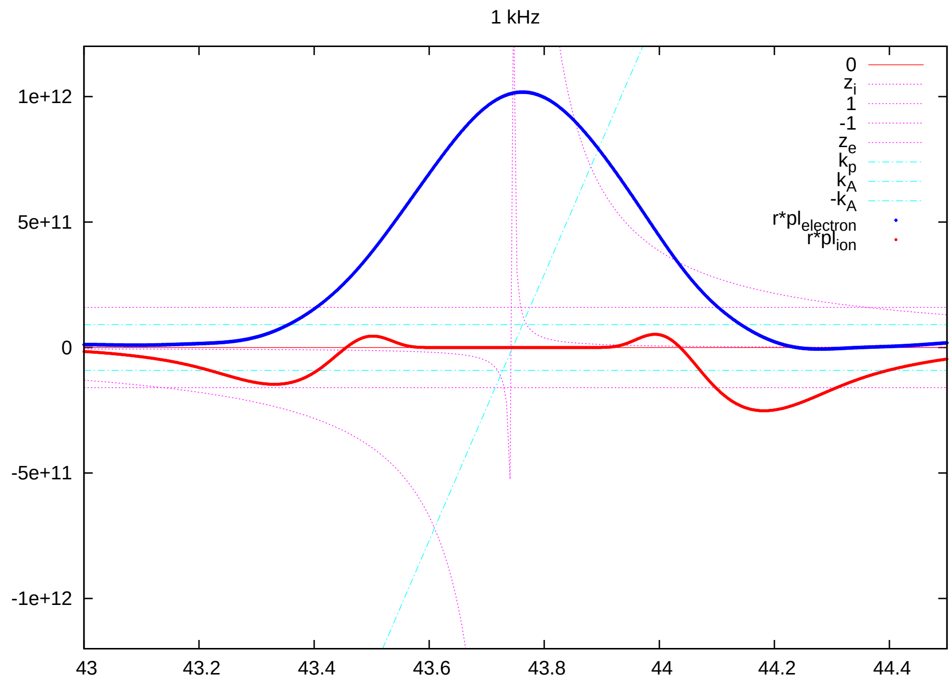

Figure 13: Electron Landau damping, ion Landau damping, local Alfvén resonance: $z_\alpha = \frac{1}{\sqrt{2}}\left[\frac{\omega/k_\parallel}{v_{t\alpha}} - \frac{V_{0\alpha}}{v_{t\alpha}}\right]$, $\frac{\omega_D}{k_\parallel} = v_{A\alpha}$.

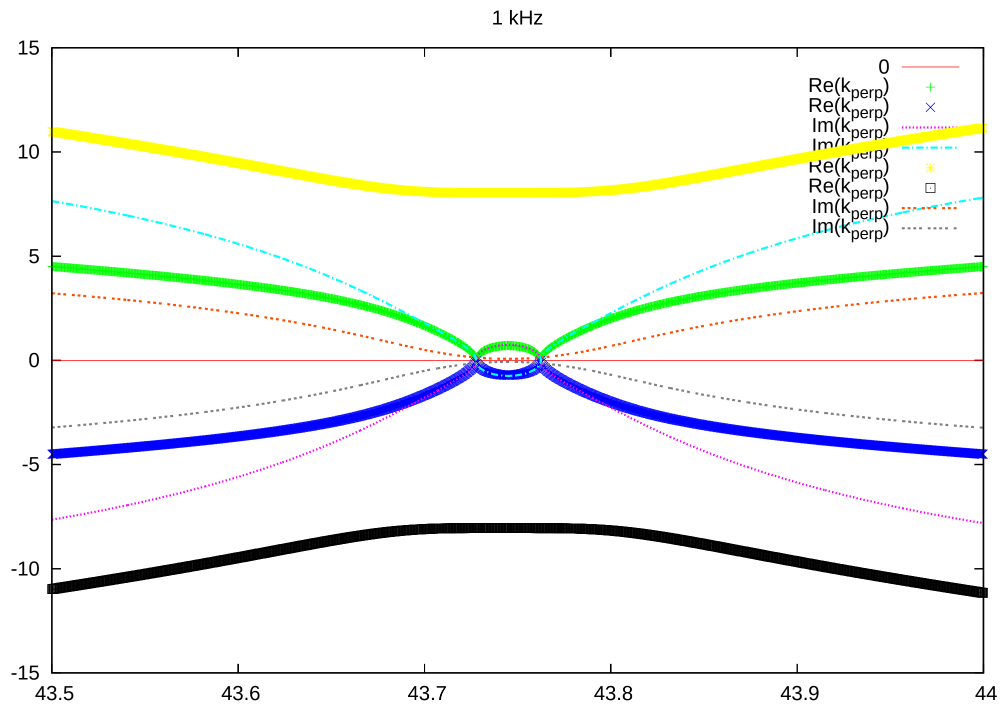

Figure 14: Slow wave dispersion ($k_\perp$).

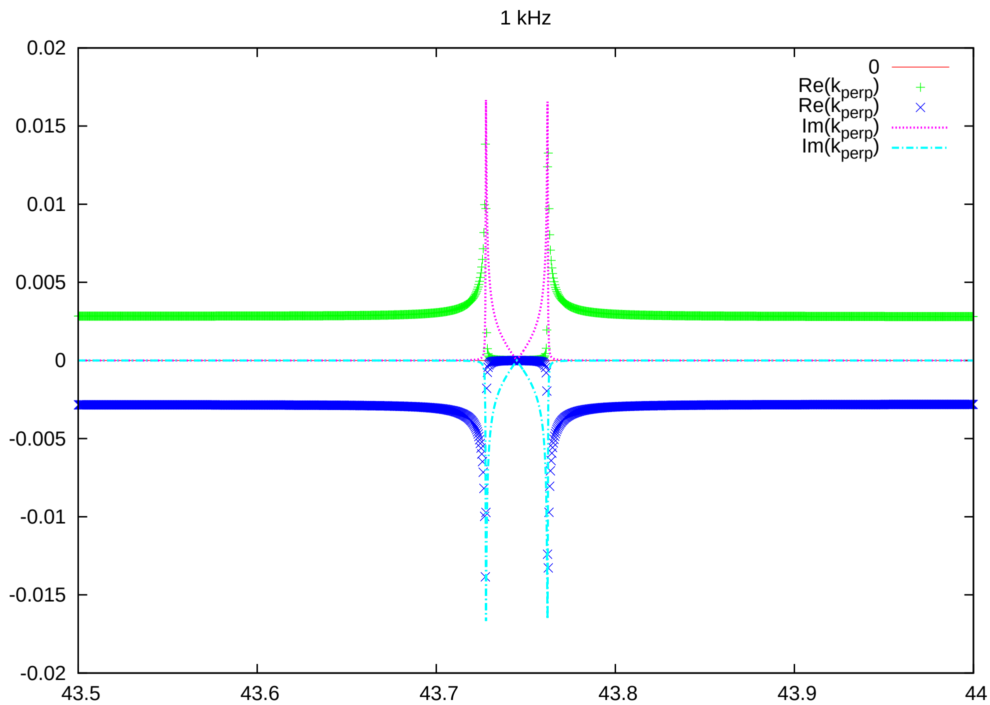

Figure 15: Fast wave dispersion ($k_\perp$).

TUG

8 Conclusions

- wave code with second order (small Larmor radius expanded) dieletric tensor
- drift waves generated within the resonant zone for temperature (also for density) gradients
- experimental evidence ?
- direct energy extraction ?

Acknowledgments

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